Solutions Algebra

J TYPE I

1. $\boldsymbol{x = (\sqrt{2}+1)^{1/3}}$

Take cube on both sides

$$x^3 = \sqrt{2}+1$$

$$\frac{1}{x^3} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \frac{\sqrt{2}-1}{1}$$

$$\frac{1}{x^3} = \sqrt{2}-1$$

$$x^3 - \frac{1}{x^3} = \sqrt{2}+1-\sqrt{2}+1 = 2$$

2. $\boldsymbol{x = 3+\sqrt{8}}$

$$x^2 = 9+8+2\times 3\sqrt{8}$$

$$x^2 = 17+6\sqrt{8}$$

$$\frac{1}{x^2} = 17-6\sqrt{8}$$

$$x^2+\frac{1}{x^2} = 17+6\sqrt{8}+17$$

$$6\sqrt{8} = 34$$

3. $\boldsymbol{x+\frac{9}{x} = 6}$

Take values of x

Let $x = 3$

$$3+\frac{9}{3} = 6$$

Prove so, $x = 3$

$$\therefore \quad x^2+\frac{9}{x^2} = 9+\frac{9}{9} = 10$$

4. $\boldsymbol{x+\frac{1}{x} = 3}$

$$\frac{x^3+\frac{1}{x}}{x^2-x+1} \text{ (divided by } x)$$

$$\frac{\frac{x^3}{x}+\frac{1}{x^2}}{\frac{x^2}{x}-\frac{x}{x}+\frac{1}{x}} = \frac{x^2+\frac{1}{x^2}}{x-1+\frac{1}{x}}$$

$$= \frac{x^2+\frac{1}{x^2}}{x+\frac{1}{x}-1}$$

$$\therefore \quad x+\frac{1}{x} = 3$$

$$\therefore \quad x^2+\frac{1}{x^2} = 9-2 = 7$$

$$\therefore \quad \frac{x^2+\frac{1}{x^2}}{x+\frac{1}{x}-1} = \frac{7}{3-1} = \frac{7}{2}$$

5. $\boldsymbol{x^4+\frac{1}{x^4} = 119 \qquad x > 1}$

$$\therefore \quad x^4+\frac{1}{x^4}+2 = 119+2 = 121$$

$$\left(x^2+\frac{1}{x^2}\right)^2 = (11)^2$$

$$x^2+\frac{1}{x^2} = 11$$

$$x^2+\frac{1}{x^2}+2 = 11+2$$

$$\left(x+\frac{1}{x}\right)^2 = 13$$

$$x+\frac{1}{x} = \sqrt{13}$$

Taking cube both sides

$$x^3+\frac{1}{x^3}+3\sqrt{13} = (\sqrt{13})^3$$

$$x^3+\frac{1}{x^3}+3\sqrt{13} = 13\sqrt{13}$$

$$x^3+\frac{1}{x^3} = 10\sqrt{13}$$

6. $\boldsymbol{x = 2-2^{1/3}+2^{2/3}}$

$$x-2 = 2^{2/3}-2^{1/3} \qquad \text{...(i)}$$

Take cube both sides

$$(x-2)^3 = (2^{2/3}-2^{1/3})^3$$

$$x^3-8-6x(x-2) = (2^{2/3})^3-(2^{1/3})^3 - 3\times 2^{2/3}.2^{1/3}(2^{2/3}-2^{1/3})$$

$$x^3-8-6x^2+12x = 2^2-2-2-3\times 2^{\frac{2+1}{3}}(x-2)$$

From equation (i)

$$\therefore \quad x^3-8-6x^2+12x = 4-2-3\times 2(x-2)$$

$$x^3+18x-6x^2-8-14 = 0$$

$$x^3+18x-6x^2-22 = 0$$

$$\therefore \quad x^3-6x^2+18x+18 = 22+18 = 40$$

7. $\boldsymbol{x + \frac{1}{x} = 2}$

(assume $x = 1$, so, $1 + 1 = 2$)

$$x^{17} + \frac{1}{x^{19}} = (1)^{17} + \frac{1}{(1)^{19}}$$
$$= 1 + 1 = 2$$

8. $\boldsymbol{n = 7 + 4\sqrt{3}}$

$$n = 4 + 3 + 4\sqrt{3}$$
$$n = (2)^2 + (\sqrt{3}^2 + 2 \times 2 \times \sqrt{3})$$
$$n = (2 + \sqrt{3})^2$$
$$\sqrt{n} = 2 + \sqrt{3}$$
$$\therefore \quad \frac{1}{\sqrt{n}} = 2 - \sqrt{3}$$
$$\sqrt{n} + \frac{1}{\sqrt{n}} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$$

9. $\boldsymbol{x + \frac{1}{x} = 2, \qquad x \neq 0}$

put $\quad x = 1$

$$1 + 1 = 2$$
$$\therefore \quad x^2 + \frac{1}{x^2} = 1 + 1 = 2$$

10. $\boldsymbol{a + \frac{1}{a} = \sqrt{3}}$

$$a^6 = -1$$
$$\therefore \quad a^6 - \frac{1}{a^6} + 2 = -1 - \frac{1}{(-1)} + 2$$
$$= -1 + 1 + 2 = 2$$

11. $\boldsymbol{x^3 + \frac{1}{x^3} = 0}$

$$\left(x + \frac{1}{x}\right)^3 - 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 0$$
$$\left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = 0$$
$$\left(x + \frac{1}{x}\right)^2 = 3\left(x + \frac{1}{x}\right)$$
$$\left(x + \frac{1}{x}\right)^2 = 3$$
$$\left(\left(x + \frac{1}{x}\right)^2\right)^2 = (3)^2$$
$$\left(x + \frac{1}{x}\right)^4 = 9$$

12. $\boldsymbol{x + \frac{1}{x} = 3}$

(Squaring both sides)

$$x^2 + \frac{1}{x^2} = 7$$

on cubing both sides

$$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$$
$$x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$
$$x^3 + \frac{1}{x^3} = 18$$
$$\therefore \quad \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) = 18 \times 7$$
$$\left(x^5 + \frac{1}{x^5}\right) + \left(x + \frac{1}{x}\right) = 126$$
$$\left(x^5 + \frac{1}{x^5}\right) + 3 = 126$$
$$\left(x^5 + \frac{1}{x^5}\right) = 123$$

13. $\boldsymbol{(x - 2)(x - 9)}$

$$= x^2 - 9x - 2x + 18$$
$$= x^2 - 11x + 18$$
$$= ax^2 + bx + c = 0$$

For minimum value $\frac{4ac - b^2}{4a}$

$$= \frac{4 \times 1 \times 18 - (-11)^2}{4 \times 1}$$
$$= \frac{72 - 121}{4} = \frac{-49}{4}$$

14. $\boldsymbol{x + \frac{1}{x} = 3}$

Squaring both sides

$$\therefore \quad x^2 + \frac{1}{x^2} + 2 = 9$$
$$x^2 + \frac{1}{x^2} = 7$$

again cubing both sides

$$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$
$$x^3 + \frac{1}{x^3} = 18$$
$$\therefore \quad \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = 7 \times 18 = 126$$
$$x^5 + \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$$
$$= 126 - 3 = 123$$

15. $\boldsymbol{x^2 + \frac{1}{x^2} = 83}$

Subtracting 2 from both sides

$$\Rightarrow \quad x^2 + \frac{1}{x^2} - 2 = 83 - 2$$
$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = 81$$

$\Rightarrow \qquad x - \frac{1}{x} = 9$

Take cube on both sides

$$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 729$$

$$x^3 - \frac{1}{x^3} - 3 \times 9 = 729$$

$$x^3 - \frac{1}{x^3} = 729 + 27 = 756$$

16. $\left(\boldsymbol{a} + \frac{\mathbf{1}}{\boldsymbol{a}}\right)^{\mathbf{2}} = \mathbf{3}$

$$a + \frac{1}{a} = \sqrt{3}$$

Take cube on both sides

$$\left(a + \frac{1}{a}\right)^3 = (\sqrt{3})^3$$

$$= a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = 3\sqrt{3}$$

$$= a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$= a^3 + \frac{1}{a^3} = 0$$

17. $\boldsymbol{x} + \frac{\mathbf{1}}{\boldsymbol{x}} = \mathbf{4}$

Squaring both sides

$$x^2 + \frac{1}{x^2} + 2 = 16$$

$$x^2 + 9\frac{1}{x^2} = 14$$

Squaring again

$$x^4 + \frac{1}{x^4} = 196 - 2 = 194$$

18. $\boldsymbol{x} - \mathbf{1}$

$$\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{96}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} + \frac{1}{x} - 1$$

$$= \frac{1}{(-1)^{99}} + \frac{1}{(-1)^{98}} + \frac{1}{(-1)^{97}} + \frac{1}{(-1)^{96}} + \frac{1}{(-1)^{95}} + \frac{1}{(-1)^{94}} + \frac{1}{(-1)} - 1$$

$$= -1 + 1 - 1 + 1 - 1 + 1 + \frac{1}{-1} - 1 = -2$$

19. $\boldsymbol{x} = \sqrt[3]{\mathbf{2} + \sqrt{\mathbf{3}}}$

$$x^3 = 2 + \sqrt{3}$$

$$\frac{1}{x^3} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$$

$\therefore \qquad x^3 + \frac{1}{x^3} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

20. $\boldsymbol{x} + \frac{\mathbf{1}}{\boldsymbol{x}} = \mathbf{2}$

Put $x = 1$

$\therefore \qquad 1 + \frac{1}{(1)} = 2$

$$2 = 2 \text{ (satisfy)}$$

$$\left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = (1+1)(1+1)$$

$$= 2 \times 2 = 4$$

21. $\boldsymbol{x}^{\mathbf{3}} + \frac{\mathbf{3}}{\boldsymbol{x}} = \mathbf{4}(\boldsymbol{a}^{\mathbf{3}} + \boldsymbol{b}^{\mathbf{3}})$ **...(i)**

$$3x + \frac{1}{x^3} = 4(a^3 - b^3) \qquad \text{...(ii)}$$

equation (i) + (ii)

$$\left(x + \frac{1}{x}\right)^3 = 8a^3$$

$$x + \frac{1}{x} = 2a \qquad \text{...(iii)}$$

$$x - \frac{1}{x} = 2b \qquad \text{...(iv)}$$

equation (iii) + (iv)

$$2(a - b) = \frac{2}{x}$$

$$a - b = \frac{2}{x}$$

$$a - b = \frac{1}{x}$$

$$a + b = x$$

$$a^2 - b^2 = 1$$

22. $\boldsymbol{x} = \mathbf{6} + \frac{\mathbf{1}}{\boldsymbol{x}}$

$$x - \frac{1}{x} = 6$$

Taking square on both sides

$$x^2 + \frac{1}{x^2} - 2 = 36$$

$$x^2 + \frac{1}{x^2} = 38$$

Again taking square on both sides

$$x^4 + \frac{1}{x^4} + 2 = (38)^2$$

$$x^4 + \frac{1}{x^4} + 2 = 1444$$

$$x^4 + \frac{1}{x^4} + 2 = 1442$$

23. $\boldsymbol{x} = \sqrt{\mathbf{3}} - \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}$ **&** $\boldsymbol{y} = \sqrt{\mathbf{3}} + \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}$

$$\frac{x^2}{y} + \frac{y^2}{x} = \frac{x^3 + y^3}{xy}$$

$$= \frac{(x + y)(x^2 - xy + y^2)}{xy}$$

$\therefore \quad x+y=\sqrt{3}-\frac{1}{\sqrt{3}}+\sqrt{3}+\frac{1}{\sqrt{3}}$

$=2\sqrt{3}$

$\therefore \quad xy=\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)$

$=3-\frac{1}{3}=\frac{8}{3}$

$\frac{(x+y)(x^2+y^2-2xy-2xy-xy)}{xy}$

$\frac{(x+y)((x+y)^2-3xy)}{xy}$

$\frac{2\sqrt{3}\left((2\sqrt{3})^2-3\times\frac{8}{3}\right)}{\frac{8}{3}}$

$\frac{2\sqrt{3}(12-8)}{\frac{8}{3}} \Rightarrow \frac{2\times3\sqrt{3}(4)}{8}=3\sqrt{3}$

24. $\boldsymbol{x-\frac{1}{x}=1}$

$\frac{x^4-\frac{1}{x^2}}{3x^2-5x-3}$

divide and multiply by x

$\frac{\frac{x^4}{x}-\frac{1}{x^3}}{\frac{3x^2}{x}+\frac{5x}{x}-\frac{3}{x}}$

$\frac{x^3-\frac{1}{x^3}}{3x-\frac{3}{x}+5} \Rightarrow \frac{x^3-\frac{1}{x^3}}{3\left(x-\frac{1}{x}\right)+5}$

$x-\frac{1}{x}=1$

Take cube on both sides

$\left(x-\frac{1}{x}\right)^3=(1)^3$

$x^3-1x^3-3\left(x-\frac{1}{x}\right)=1$

$x^2-\frac{1}{x^3}-3=1$

$x^3-\frac{1}{x^3}=4$

$=\frac{x^3-\frac{1}{x^3}}{3\left(x-\frac{1}{x}\right)+5}=\frac{4}{3\times1+5}=\frac{4}{8}=\frac{1}{2}$

25. $\boldsymbol{x^2+\frac{1}{x^2}=66}$

$x^2+\frac{1}{x2}-2=66-2$

$\left(x-\frac{1}{x}\right)^2=64$

$\left(x-\frac{1}{x}\right)^2=(8)^2$

$x-\frac{1}{x}=\pm8$

$\frac{x^2-1+2x}{x}=\frac{\frac{x^2}{x}-\frac{1}{x}+\frac{2x}{x}}{\frac{x}{x}}$

$\frac{\left(x-\frac{1}{x}\right)+2}{1}$

When $\quad x-\frac{1}{x}=+8$

Then $\quad \left(x-\frac{1}{x}\right)+2=8+2=10$

When $\quad x-\frac{1}{x}=-8$

$-8+2=-6$

$\therefore (10, -6)$

26. $\boldsymbol{x+\frac{2}{x}=1}$

$x^2+2=x$

$x^2-x=-2$

$x-x^2=2$

$\therefore \quad \frac{x^2+x+2}{x^2(1-x)}=$ divide & multiply by x

$\therefore \quad \frac{\frac{x^2}{x}+\frac{x}{x}+\frac{2}{x}}{\frac{x^2}{x}(1-x)}=\frac{x+\frac{2}{x}+1}{x(1-x)}$

$\frac{x+\frac{1}{x}+1}{x-x^2}=\frac{1+1}{2}=1$

27. $\boldsymbol{x+\frac{1}{x}=2}$ **...(i)**

Squaring both sides

$x^2+\frac{1}{x^2}+2=4$

$x^2+\frac{1}{x^2}=+2$

Cubing equation (i)

$x^3+1x^3+3\left(x+\frac{1}{x}\right)=8$

$x^3+\frac{1}{x^3}+6=8$

$$x^3 + \frac{1}{x^3} = 2$$

$$\therefore \quad \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = 2 \times 2 = 4$$

28. $\boldsymbol{x + \frac{1}{x} = 5}$

$\therefore$ take cube on both sides

$$\left(x + \frac{1}{x}\right)^3 = (5)^3$$

$$x^3 + \frac{1}{x^3} + 3 \times 5 = 125$$

$$x^3 + \frac{1}{x^3} = 110$$

$\therefore$ Squaring both sides

$$\left(x^3 + \frac{1}{x^3}\right)^2 = (110)^2$$

$$x^6 + \frac{1}{x^6} + 2 = 12100$$

$$x^6 + \frac{1}{x^6} = 12100 - 2 = 12098$$

29. $\boldsymbol{\left(a + \frac{1}{a}\right)^2 = 3}$

$$a + \frac{1}{a} = \sqrt{3}$$

Cube on both sides

$$\left(a + \frac{1}{a}\right)^3 = (\sqrt{3})^3$$

$$a^3 + 1a^3 + 3\sqrt{3} = 3\sqrt{3}$$

$$a^3 + \frac{1}{a^3} = 0$$

30. $\boldsymbol{x + \frac{1}{x} = 3}$

$$\frac{3x^2 + 3 - 4x}{x^2 + 1 - x} = \frac{\frac{3x2}{x} + \frac{3}{x} - 4}{\frac{x^2}{x} + \frac{1}{x} - \frac{x}{x}} = \frac{3\left(x + \frac{1}{x}\right) - 4}{\left(x + \frac{1}{x}\right) - 1}$$

$$= \frac{3 \times 3 - 4}{3 - 1} = \frac{9 - 4}{3 - 1} = \frac{5}{2}$$

31. $\boldsymbol{x + \frac{1}{x} = 2\frac{1}{12} = \frac{25}{12}}$

$$x^2 + \frac{1}{x^2} + 2 = \frac{675}{144}$$

$$x^2 + \frac{1}{x^2} = \frac{625}{144} - 2$$

$$x^2 + \frac{1}{x^2} = \frac{625 - 288}{144}$$

$$x^2 + \frac{1}{x^2} = \frac{337}{144}$$

$$x^2 + \frac{1}{x^2} - 2 = \frac{337}{144} - 2$$

$$\left(x - \frac{1}{x}\right)^2 = \frac{337 - 288}{144} = \frac{49}{144}$$

$$x - \frac{1}{x} = \frac{7}{12}$$

$$\therefore \quad \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = \frac{25}{12} \times \frac{7}{12} = \frac{175}{144}$$

$$\therefore \quad \left(x^2 - \frac{1}{x^2}\right) = \frac{175}{144}$$

$$\therefore \quad x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$$

$$= \frac{175}{144} \times \frac{337}{144} = \frac{58975}{20736}$$

32. $\boldsymbol{t^2 - 4t + 1 = 0}$

$$\frac{t^2 + 1}{t} = \frac{4t}{t}$$

$$t + \frac{1}{t} = 4$$

[take cube both sides]

$$t + \frac{1}{t} + 3t\frac{1}{t}\left(t + \frac{1}{t}\right) = 64$$

$$t^3 + \frac{1}{t^3} = 64 - 12 = 52$$

$$t^3 + \frac{1}{t^3} = 52$$

33. Given

$$\frac{x^{24} + 1}{x^{12}} = 7$$

$$\frac{x^{24} + 1}{x^{12}} = \frac{x^{24}}{x^{12}} + \frac{1}{x^{12}}$$

$$x^{12} + \frac{1}{x^{12}} = 7$$

Cubing both sides

$$\left(x^{12} + \frac{1}{x^{12}}\right)^3 = 7^3$$

$$x^{36} + \frac{1}{x^{36}} + \frac{3 \times x^{12} \times 1}{x^{12}}\left(x^{12} + \frac{1}{x^{12}}\right) = 343$$

$$x^{36} + \frac{1}{x^{36}} + 3 \times 7 = 343$$

$$x^{36} + \frac{1}{x^{36}} = 343 - 21$$

$$x^{36} + \frac{1}{x^{36}} = \frac{x^{72} + 1}{x^{36}} = 322$$

34. Given $\boldsymbol{P = 99}$

Find $P(P^2 + 3P + 3) = ?$

to put value in equaiton

$99((99)^2 + (3 \times 99) + 3)$

$(100 - 1)[(100 - 1)^2 + [3 \times (100] - 1] + 3)$

$(100 - 1)[10000 + 1 - 200 + 300 - 3 + 3]$

$(100-1)(10000+100+1)$
$(100-1)(10101)$
99×10101
99 99 99

35. **Given,** $\boldsymbol{4a - \frac{4}{a} + 3 = 0}$

Find $a^3 - \frac{1}{a^3} + 3 = ?$

$$4a - \frac{4}{a} = -3$$

$$a - \frac{1}{a} = \frac{-3}{4}$$

$$\left(a - \frac{1}{a}\right)^3 = \left(\frac{-3}{4}\right)^3 \text{ (cubing both sides)}$$

$$a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a}\left(a - \frac{1}{a}\right) = \frac{-27}{64}$$

$$a^3 - \frac{1}{a^3} - 3 \times \left(-\frac{3}{4}\right) = \frac{-27}{64}$$

$$a^3 - \frac{1}{a^3} = \frac{-27}{64} - \frac{9}{4}$$

$$a^3 - \frac{1}{a^3} + 3 = \frac{-27}{64} = -\frac{9}{4} + 3$$

$$\frac{192-171}{64} = a^3 - \frac{1}{a^3} + 3 = \frac{21}{64}$$

36. **Given** $\boldsymbol{x + \frac{1}{x} = 2}$ **...(i)**

The value of $x^{12} - \frac{1}{x^{12}} = ?$

If $x = 1 \Rightarrow x + \frac{1}{x} = 2$

$$1 + 1 = 2$$

Then, $x^{12} - \frac{1}{x^{12}}$

$1^{12} - \frac{1}{1^{12}}$

$$1 - 1 = 0$$

37. **Given** $\boldsymbol{x + \frac{1}{x} = 1}$ **...(i)**

Find $\frac{x^2 + 3x + 1}{x^2 + 7x + 1} = ?$

From equation (i)

$$x + \frac{1}{x} = 1$$

$$x^2 + 1 = x$$

$\frac{(x^2+1)+3x}{(x^2+1)+7x}$

$\frac{x+3x}{x+7x} \Rightarrow \frac{4x}{8x} = \frac{1}{2}$

38. $\boldsymbol{x + \frac{1}{x} = 2}$

Find $x^7 + \frac{1}{x^5} = 2$

$$x + \frac{1}{x} = 2$$

$$\downarrow \;\; \downarrow \qquad \text{Let } x = 1$$

$$1 \;\; 1$$

To, put value in question,

$$x^7 + \frac{1}{x^5} \Rightarrow 1^7 + \frac{1}{1^5}$$

$$1 + 1 = 2$$

39. $\boldsymbol{\frac{2p}{p^2 + 2p + 1} = \frac{1}{4}}$

$$\frac{2}{p - 2 + \frac{1}{p}} = \frac{1}{4}$$

(Divide p both in nu. & de.)

$$p + \frac{1}{p} - 2 = 8$$

$$p + \frac{1}{p} = 10$$

40. **According to the quesiton If** $\boldsymbol{x = 5}$

$x^2 - 2 + \frac{1}{x2}$

$$\left(x - \frac{1}{x}\right)^2 = \left(5 - \frac{1}{5}\right)^2$$

$$\left(\frac{24}{5}\right)^2 = \frac{576}{25}$$

41. $\boldsymbol{x^2 - 4x - 1 = 0}$

$$x - \frac{1}{x} = 4$$

$$\left(x - \frac{1}{x}\right)^2 = 4^2$$

(squaring both sides)

$$x^2 + \frac{1}{x^2} = 18$$

42. $\boldsymbol{x + \frac{1}{x} = 3}$

(Cube both sides Formula)

$$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$$

$$x^3 + \frac{1}{x^3} = 18$$

$$x + \frac{1}{x} = 3 \text{ (squaring both sides)}$$

$$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 9$$

$$x^2 + \frac{1}{x^2} = 7$$

Put in formula,

43. $\boldsymbol{a + \frac{1}{a} = \sqrt{3}}$

$$a^3 + \frac{1}{a^3} = (\sqrt{3})^3 - 3\sqrt{3}$$

$$a^6 = -1$$

$$a^2 + \frac{1}{a^2} = 1$$

then $$a^{52} + \frac{1}{a^{52}} = \frac{a^{54}}{a^2} + \frac{a^2}{a^{54}}$$

$$= \frac{(a^6)^9}{a^2} + \frac{a^2}{(a^6)^9}$$

$$= -\frac{1}{a^2} - a^2 = -\left(a^2 + \frac{1}{a^2}\right)$$

then $a^{52} + \frac{1}{a^{52}} = -1$

44. **If** $\boldsymbol{x^2 + \frac{1}{x^2} = 1}$

Then, $$x + \frac{1}{x} = \sqrt{3}$$

$$x^3 + \frac{1}{x^3} = (\sqrt{3})^3 - 3\sqrt{3} = 0$$

$$x^6 = -1, \quad \text{or} \quad x^6 + 1 = 0$$

Then $x^{102} + x^{96} + x^{90} + x^{84} + x^{78} + x^{72} + 5$

$x^{96}(x^6 + 1) + x^{84}(x^6 + 1) + x^{72}(x^6 + 1) + 5 = 5$

45. $\boldsymbol{x^3 - 7x^2 + 11x - 5 \geq 0}$

$x^3 - 5x^2 - 2x^2 + 10x + x - 5 \geq 0$

$x^2(x - 5) - 2x(x - 5) + 1(x - 5) \geq 0$

$(x - 5)(x^2 - 2x + 1) \geq 0$

$(x - 5)(x - 1)^2 \geq 0$

$(x - 5)(x - 1)(x - 1) \geq 0$

So, $x = 1 \quad \& \quad 5$

Equation satisfies at both the values, but the minium value of these two

$$x = 1$$

46. $\boldsymbol{x^2 + \frac{1}{x^2} = 1}$

$$\left(x + \frac{1}{x}\right)^2 = 3$$

$$x + \frac{1}{x} = \sqrt{3}$$

$$x^3 + \frac{1}{x^3} = 0 \quad = x^6 + 1 = 0$$

Now, $x^{18} + x^{12} + x^6 + 1$

$x^{12}(x^6 + 1) + (x^6 + 1) = x^{12} \times 0 + 0 = 0$

47. $\boldsymbol{3x^2 + 5x + 3 = 0}$

$$3x^2 + 3 = 5x$$

divide by $3x$ both sides

$$x + \frac{1}{x} = \frac{-5}{3} \text{ then } x^3 + \frac{1}{x^3}$$

$$= \left(\frac{-5}{3}\right)^3 - 3 \times \left(\frac{-5}{3}\right)$$

$$= \frac{-125}{27} + 5 = \frac{10}{27}$$

J TYPE 2

1. $\boldsymbol{\frac{2a + b}{a + 4b} = 3}$

$$2a + b = 3(a + 4b)$$

$$2a + b = 3a + 12b$$

$$-a = 11b$$

$$a = -11b$$

$\therefore$ $$\frac{a + b}{a + 2b}$$

$$\frac{-11b + b}{-11b + 2b} = \frac{-10b}{-9b} = \frac{10}{9}$$

2. $\boldsymbol{x \otimes y = 3x + 2y}$

$$(2 \otimes 3) = 3 \times 2 + 2 \times 3$$

$$= 6 + 6 = 12$$

$$(3 \otimes 4) = 3 \times 3 + 2 \times 4 = 9 + 8 = 17$$

$\therefore$ $(2 \otimes 3) + (3 \otimes 4) = 12 + 17 = 29$

3. $\boldsymbol{10^{0.48} = x}$

$$10^{0.70} = y$$

and $$x^z = y^2$$

$\therefore$ $$(10^{0.48})^x = (10^{0.70})^2$$

$10^{0.48z} = 10^{1.40}$ (if $a^x = z^y$, fi base equal power are equal ($x = y$)

$\therefore$ $$0.48z = 1.40$$

$$z = \frac{140}{48} = \frac{35}{12} = 2.9$$

4. $\boldsymbol{x * Y = x^2 + y^2 - xy}$

$$9 * 11 = (9)^2 + (11)^2 - 11 \times 9$$

$$81 + 121 - 99 = 103$$

5. $\boldsymbol{\frac{2x - y}{x + 2y} \times \frac{1}{2}}$ **(cross Multiply)**

$$4x - 2y = x + 2y$$

$$3x = 4y$$

$$x : y = 4 : 3$$

$$\frac{3x - y}{3x + y} = \frac{3 \times 4 - 3}{3 \times 4 + 3}$$

$$= \frac{12 - 3}{12 + 3} = \frac{9}{15} = \frac{3}{5}$$

6. $\boldsymbol{1.5x = 0.04y}$

$$\frac{x}{y} = \frac{0.04}{1.5} = \frac{4}{100} \times \frac{10}{15} = \frac{2}{75}$$

$\therefore \quad \dfrac{y^2-x^2}{y^2-x^2+2xy} = \dfrac{(y-x)(y+x)}{(y+x)^2}$

$$\frac{y-x}{y+x} = \frac{75-2}{75+2} = \frac{73}{77}$$

7. $\boldsymbol{x^2+y^2+\dfrac{1}{x^2}+\dfrac{1}{y^2}=4}$

Take $\quad x = y = 1$

$$1+1\frac{1}{1}+\frac{1}{1} = 4$$

Hence $\quad x^2+y^2 = 1+1 = 2$

8. $\boldsymbol{a^2+b^2+2b+4a+5=0}$

$a^2+b^2+2b+4a+4+1=0$

$a^2+4a+4+b^2+2b+1=0$

$(a+2)^2+(b+1)^2=0$

$a+2=0 \qquad a=-2$

$b+1=0 \qquad b=-1$

$\dfrac{a-b}{a+b} \Rightarrow \dfrac{-2+1}{-211}$

$\dfrac{-1}{-3} = \dfrac{1}{3}$

9. $\boldsymbol{x^2+y^2-4x-4y+8=0}$

$x^2+4-4x+y^2+4-4y=0$

$(x-2)^2+(y-2)^2=0$

$x-2=0, \qquad y-2=0$

$x=2 \qquad y=2$

$\therefore \quad x-y = 2-2 = 0$

10. $\boldsymbol{\dfrac{1}{x+y} = \dfrac{1}{x}+\dfrac{1}{y}}$

$$\frac{1}{x+y} = \frac{x+y}{xy}$$

$$xy = (x+y)^2$$

$$x^2+y^2+2xy = xy$$

$$x^2+y^2+xy = 0$$

$\therefore \quad x^3-y^3 = (x-y)(x^2+y^2+xy)$

$$(x^3-y^3) = (x-y)\times 0 = 0$$

11. $\boldsymbol{xy(x+y)=1}$

$$x+y = \frac{1}{xy}$$

Cubing both sides

$$(x+y)^3 = \frac{1}{x^3y^3}$$

$$x^3+y^3+3xy(x+y) = \frac{1}{x^3y^3}$$

$$x^3+y^3+3 = \frac{1}{x^3y^3}$$

$\because \quad \left(x+y = \dfrac{1}{xy}\right)$

12. $\boldsymbol{x-y=2, xy=24}$

$$x^2+y^2-2xy = 4$$

$$x^2+y^2-2\times 24 = 4$$

$$x^2+y^2 = 4+48 = 52$$

13. $\boldsymbol{x^3+y^3=35}$

$$x+y=5$$

Take cube on both sides,

$$(x+y)^3 = (5)^3$$

$$x^3+y^3+3xy(x+y) = 125$$

$$35+3xy(5) = 125$$

$$15xy = 125-35$$

$$15xy = 90$$

$$xy = 6$$

$\therefore \quad \dfrac{1}{x}+\dfrac{1}{y} = \dfrac{x+y}{xy} = \dfrac{5}{6}$

14. $\boldsymbol{(a^2+b^2)^3 = (a^3+b^3)^2}$

$a^6+b^6+3a^2b^2(a^2+b^2) = a^6+b^6+2a^3b^3$

$a^6+b^6+3a^4b^2+3a^2b^4 = a^6+b^6+2a^3b^3$

$3a^4b^2+3a^2b^4 = 2a^3b^4$

$a^2b^2(3a^2+3b^2) = 2a^3b^3$

$3a^2+3b^2 = 2ab$

$\dfrac{a^2+b^2}{ab} = \dfrac{2}{3}$

$\dfrac{a}{b}+\dfrac{b}{a} = \dfrac{2}{3}$

$\dfrac{a}{b}+\dfrac{b}{a} = \dfrac{2}{3}$

15. $\boldsymbol{x^2+y^2+1=2x}$

$$x^2-2x+1+y^2=0$$

$$(x-1)^2+y^2=0$$

If $\quad A^2+B^2=0$

[As power are even it can possible only when $A=0$ & $B=0$]

$\therefore \quad x-1=0$

$$x=1$$

$$y=0$$

$\therefore \quad x^3+y^5 = 1+0 = 1$

16. $\boldsymbol{a^3+b^3=9}$

$$a+b=3$$

Assume values, $a=2, \quad b=1$

$\therefore \quad (2)^3+1=9$

$$2+1=3$$

$\therefore \quad \dfrac{1}{a}+\dfrac{1}{b} = \dfrac{1}{2}+1 = \dfrac{3}{2}$

17. **Given**

$$5x+9y=5 \qquad \ldots\text{(i)}$$

$$125x^3+729y^3=120 \qquad \ldots\text{(ii)}$$

From equation (i) cubing both sides

$$(5x+9y)^3 = 5^3$$

$$125x^3 + 729y^3 + 3 \times 5x + 9y\,(5x + 9y) = 125$$
$$128x^3 + 729y^3 + 135xy \times 5 = 125$$
$$120 + 135xy \times 5 = 125$$
$$135xy \times 5 = 5$$
$$xy = \frac{1}{135}$$

therefore product of x & $y = \frac{1}{135}$

19. $\boldsymbol{999x + 888y = 1332}$

$$\frac{888x + 999y - 555}{1887(x+y) = 1887}$$
$$x + y = 1$$

20. According to the question,

$$x = \frac{1}{2+\sqrt{3}},\ y = \frac{1}{2-\sqrt{3}}$$
$$x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}},$$
$$y = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$
$$x = 2-\sqrt{3},\ y = 2+\sqrt{3}$$
$$8xy\,(x^2 + y^2) = 8(2-\sqrt{3})(x+\sqrt{3})$$
$$[(2-\sqrt{3})^2 + (2+\sqrt{3})^2]$$
$$8 \times 1[7 - 2\sqrt{3} + 7 + 2\sqrt{3}] = 112$$

21. According to the question

$$\frac{1}{x+y} + \frac{1}{x-y}$$
$$\frac{x - y + x + y}{x^2 - y^2} = \frac{2x}{x^2 - y^2}$$

22. $\boldsymbol{x : y = 3 : 5}$

$$x - y = -2$$
$$\frac{x}{y} = \frac{3}{5}$$
$$x - y = 3 - 5 = -2 \rightarrow -2$$
$$x = 3, y = 5$$
$$x + y = 3 + 5 = 8$$

23. According to the question

$$x = 1 + \sqrt{2} + \sqrt{3}$$
$$y = 1 + \sqrt{2} - \sqrt{3}$$
$$\frac{x^2 + 4xy + y^2}{x+y}$$
$$\frac{(x+y)^2 + 2xy}{x+y}$$
$$x + y = 2 + 2\sqrt{2}$$
$$xy = (1+\sqrt{2})^2 - (\sqrt{3})^2$$
$$= 3 + 2\sqrt{2} - 3$$
$$= 2\sqrt{2}$$
$$\frac{(2+2\sqrt{2})^2 + 4\sqrt{2}}{2+2\sqrt{2}}$$
$$\frac{4 + 8 + 8\sqrt{2} + 4\sqrt{2}}{2+2\sqrt{2}}$$
$$\frac{12 + 12\sqrt{2}}{2+2\sqrt{2}}$$
$$= \frac{12(1+\sqrt{2})}{2(1+\sqrt{2})} = 6$$

24. $\boldsymbol{a - 6b = 3}$

after cubing both sides, we get

$$(a - 6b)^3 = (3)^3$$
$$a^3 - 216b^3 - 18ab(3) = 27$$
$$a^3 - 216b^3 - 54ab = 27$$

J TYPE 3

1. **A : B : C**

2 : 3 : 4

$\therefore \quad \frac{A}{B} : \frac{B}{C} : \frac{C}{A}$ (Multiply with ABC)

$\therefore \quad \frac{A \times ABC}{B} : B^2A : BC^2$

$(2)^2 \times 4 : (3)^2 \times 2 : 3 \times (4)^2$

16 : 18 : 48

8 : 9 : 24

Alternate :

$$\frac{A}{B} : \frac{B}{C} : \frac{C}{A} = \frac{2}{3} : \frac{3}{4} : \frac{4}{2}$$
$$\frac{8,\ 9,\ 24}{12} \Rightarrow 8 : 9 : 24$$

2. $\boldsymbol{x + y = 2z}$

$$x - z = z - y$$
$$x - z \Rightarrow\ -(y - z) \qquad \ldots\text{(i)}$$
$$\frac{x}{x-z} + \frac{z}{y-z} = \frac{x}{x-z} - \frac{z}{x-z}$$
$$= \frac{x-z}{x-z} = 1$$

3. $\boldsymbol{a^3 b = abc = 180}$

or $\quad a^2 = c = 180$

$\therefore \quad c = 180$

4. $\boldsymbol{ax^2 + bx + c = a(x - p)^2}$

$$ax^2 + bx + c = a(x^2 + p^2 - 2px)$$
$$ax^2 + bx + c = ax^2 + ap^2 - 2apx$$

Comparing cofficients of x^2 and x

$$b = -2ap$$
$$p = \frac{-b}{2a} \qquad \ldots\text{(i)}$$

and $\quad c = ap^2$

$$c = a \times \frac{b^2}{4a^2} \qquad \text{(from (i))}$$

$$4ac = b^2$$

5. **Put $a = b = 1$ and $c = -2$**

we get $a + b + c = 1 + 1 - 2 = 0\,0 = 0$

(satisfy)

$$\therefore \quad \frac{a^2 + b^2 + c^2}{a^2 - bc} = \frac{(1)^2 + (1)^2 + (-2)^2}{(1)^2 - (1)(-2)}$$

$$= \frac{6}{3} = 2$$

6. $\boldsymbol{\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} + 3 = 1}$

Adding 3 on both sides

$$\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} + 3 = 1 + 3$$

$$\frac{a}{1-a} + 1 + 1\frac{b}{1-b} + 1 + \frac{c}{1-c} + 1 = 4$$

$$\frac{a+1-a}{1-a} + \frac{b+1-b}{1-b} + \frac{c+1-c}{1-c} = 4$$

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

7. $\boldsymbol{(x-3)^2 + (y-5)^2 + (z-4)^2 = 0}$

$$\therefore \quad (x-3)^2 = 0 \qquad x = 3$$

$$(y-5)^2 = 0 \qquad y = 5$$

$$(z-4)^2 = 0 \qquad z = 4$$

$$\therefore \quad \frac{x^2}{9} + \frac{y^2}{25} + \frac{c^2}{16}$$

$$\frac{9}{9} + \frac{25}{25} + \frac{16}{16} = 3$$

8. $\boldsymbol{\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0}$

$$= 4 - \frac{3}{x} + 4 - \frac{3}{y} + 4 - \frac{3}{z} = 0$$

$$12 - 3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$$

$$-3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = -12$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 4$$

9. $\boldsymbol{x^2 = y + z}$

$y^2 = z + x$

$z^2 = x + y$

$x^2 + x = x + y + z$

Adding x on both sides

$$x(x+1) = x + y + z$$

$$\frac{1}{(x+1)} = \frac{x}{x+y+z}$$

Similarly

$$\frac{1}{y+1} = \frac{y}{x+y+z} \text{ and } \frac{1}{z+1} = \frac{z}{x+y+z}$$

$$\therefore \quad \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{1+z} = \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z}$$

$$= \frac{x+y+z}{x+y+z} = 1$$

10. $\boldsymbol{xy + yz + zx = 0}$

$$\therefore \quad xy + zx = -yz$$

$$xy + yz = -zx$$

$$yz + zx = -xy$$

$$\therefore \frac{1}{x^2 - yz} + \frac{1}{y^2 - zx} + \frac{1}{z^2 - xy}$$

Puting values of $-yz, -zx, -xy$ from above

$$\Rightarrow \frac{1}{x^2 + (xy + zx)} + \frac{1}{y^2 + (xy + yx)} + \frac{1}{z^2 + (yz + zx)}$$

$$\Rightarrow \frac{1}{x(x+y+z)} + \frac{1}{y(x+y+z)} + \frac{1}{z(x+y+z)}$$

$$\Rightarrow \frac{1}{(x+y+z)}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

$$\Rightarrow \frac{1}{(x+y+z)}\left(\frac{zy + xz + xy}{xyz}\right)$$

$$\Rightarrow \frac{1}{x+y+z} \times 0 = 0$$

11. $\boldsymbol{\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}}$

Go through options 'd' take $x = y = z$

$$\frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2} = \frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2}$$

$\therefore$ Option d is right

12. $\boldsymbol{x = a - b}$

$y = b - c$

$z = c - a$

$$x + y + z = a - b + b - c + c - a = 0$$

$$\therefore \quad x^3 + y^3 + z^3 - 3xyz = (x + y + z)$$

$$(x^2 + y^2 + z^2 - xy - yz - zx) = 0$$

13. $\boldsymbol{a^2 + b^2 + c^2 = 2(a - b - c) - 3}$

$a^2 + b^2 - c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$

$a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$

$(a-1)^2 + (b+1)^2 + (c+1)^2 = 0$

$$a - 1 = 0 \qquad a = 1$$

$$b + 1 = 0 \qquad b = -1$$

$$c + 1 = 0 \qquad c = -1$$

$$= 4 \times 1 - 3 \times -1 + 5 \times -1$$

$$= 4 + 3 - 5 = 2$$

14. $\boldsymbol{ab + bc + ca = 0}$

$$-bc = (ab + ca)$$
$$-ac = ab + bc$$
$$-ab = bc + ca$$
$$\frac{1}{a^2 - bc} + \frac{1}{b^2 - ac} + \frac{1}{c^2 - ab}$$
$$\frac{1}{a^2 + ab + ca} + \frac{1}{b^2 + ab + bc} + \frac{1}{c^2 + bc + ca}$$
$$\frac{1}{a(a+b+c)} + \frac{1}{b(a+b+c)} + \frac{1}{c(a+b+c)}$$
$$\frac{1}{a+b+c}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$
$$\frac{1}{a+b+c}\left(\frac{ab+bc+ca}{abc}\right) = 0$$
$(\because ab + bc + ca = 0$ given)

15. $\boldsymbol{\frac{b-c}{a} + \frac{a-c}{b} + \frac{a-b}{c} = 1}$

$$a - b + c \neq 0$$
Let $\quad b = c$
$$\therefore \quad \frac{b-b}{a} + \frac{a+b}{b} + \frac{a-b}{b} = 1$$
$$0 + \frac{a}{b} + 1 + \frac{a}{b} - 1 = 1$$
$$\frac{a}{b} + \frac{a}{b} = 1$$
$$\frac{1}{b} + \frac{1}{b} = \frac{1}{a}$$
we take $\quad b = c$
$$\therefore \quad \frac{1}{b} + \frac{1}{c} = \frac{1}{a}$$

16. $\boldsymbol{a + b + c + d = 4}$

$$\frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-a)(1-b)(1-c)}$$
$$+ \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-b)(1-c)(1-d)}$$
Put $\quad a = 0, \quad b = 0$ and $c = 2$ and $d = 2$
$$a + b + c + d = 0 + 0 + 2 + 2$$
$$= 4 + 4 \text{ (satisify)}$$

17. $\boldsymbol{3(a^2 + b^2 + c^2) = (a + b + c)^2}$

By options $\quad a = b = c$
$$3(a^2 + a^2 + a^2) = 9a^2$$
$$9a^2 = 9a^2$$

18. $\boldsymbol{(a + b + c)^2 + (b + c - a)^2 + (c + a - b)^2 = ?}$

$$a + b + c = 0 \quad \text{(given)}$$
$$a + b = -c$$
$$b + c = -a$$
$$a + c = -b$$
$$(a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2$$
$$(-c - c)^2 + (-a - a)^2 + (-b - b)^2$$
$$(-2c)^2 + (-2a)^2 + (-2b)^2$$
$$4c^2 + 4a^2 + 4b^2$$
$$4(a^2 + b^2 + c^2)$$

19. According to the question,

$$\because \quad a + b - c = 14$$
Find $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4 = ?$
$$a + b - c = 14$$
Let $\quad a = 7,\ b = 7,\ c = 0$
$$\therefore \quad 2 \times 7^2 \times 0 + 2 \times 0^2 \times 7^2 + 2 \times 7^2 \times 7^2 - 7^4$$
$$-7^4 - 0^4 = 0$$

20. Here, $\boldsymbol{x = 332, y = 333, z = 335}$

Find $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)$
$$[(x-y)^2 + (y-z)^2 + (z-x)]$$
$$= \left(\frac{332 + 333 + 335}{2}\right)[(333 - 332)^2 + (335 - 333)^2$$
$$+ (335 - 332)^2]$$
$$= \frac{1000}{2}(1^2 + 2^2 + 3^2)$$
$$= \frac{1000}{2}(14) = 7000$$

21. According to the question,

$$a^2 + b^2 + c^2 = ab + bc + ca$$
Put $\quad a = 1, \quad b = 1, \quad c = 1$
$$\therefore \quad 1^2 + 1^2 + 1^2 = 1 \times 1 + 1 \times 1 + 1 \times 1$$
$$1 + 1 + 1 = 1 + 1 + 1$$
$$3 = 3 \text{ [Satisfy]}$$
$$\therefore \quad \frac{a+c}{b} = \frac{1+1}{1} = 2$$

23. $\boldsymbol{a^2 + b^2 + c^2 = ab + bc + ca}$

Let $a = b = c = 1$
$$a^2 + b^2 + c^2 = ab + bc + ca$$
$$1^2 + 1^2 + 1^2 = 1 \times 1 + 1 \times 1 + 1 \times 1$$
$$3 = 3$$
to find $\quad \frac{a+c}{b} = ?$
$$\frac{1+1}{1} = 2$$

25. $\boldsymbol{\frac{x}{a} = b - c : \frac{y}{b} = c - a; \frac{z}{c} = a - b}$

$$\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = 3\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(\frac{z}{c}\right)$$
$$\left(\because \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0\right) = \frac{3xyz}{abc}$$

26. $\boldsymbol{\frac{x}{y} = \frac{a+2}{a-2}}$

$$\frac{x^2}{y^2} = \frac{(a+2)^2}{(a-2)^2}$$
Applying componendo & Dividendo
$$\frac{x^2 - y^2}{x^2 + y^2} = \frac{(a+2)^2 - (a-2)^2}{(a+2)^2 + (a-2)^2}$$

$$= \frac{8a}{2a^2+8} = \frac{4a}{a^2+4}$$

27. $\dfrac{(\boldsymbol{a}+\boldsymbol{y}+\boldsymbol{z})^2}{\boldsymbol{x}^2+\boldsymbol{y}^2+\boldsymbol{z}^2}$

Assume $x = y = z = 1$

$$\frac{(1+1+1)^3}{1+1+1} = (3)$$

28. $(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})^2 = \boldsymbol{a}^2+\boldsymbol{b}^2+\boldsymbol{c}^2+\boldsymbol{2}$

$(ab+bc+ca)$

$$676 = a^2+b^2+c^2+2(109)$$

$$ah2+b^2+c^2 = 458$$

29. If $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}=\boldsymbol{0}$

put $a = 1\, b = 1$ and $c = -2$

$$\frac{a^2+b^2+c^2}{ab+bc+ca} = \frac{1+1+4}{1-2-2} = \frac{6}{-3}$$

$$= -2$$

TRIGNOMENTY SOLUTION

1. **Put,** $\theta = 60^\circ$

$\Rightarrow \quad \cos\theta > \cos^2\theta$

$\Rightarrow \quad \cos 60^\circ > \cos^2 60^\circ$

$\Rightarrow \quad \frac{1}{2} > \frac{1}{4}$

$\cos\theta > \cos^2\theta$

2. $\mathbf{7\sin^2\theta + 3\cos^2\theta = 4}$

$7\sin^2\theta + 3(1-\sin^2\theta) = 4$

$7\sin^2\theta + 3(1-\sin^2\theta) = 4$

$7\sin^2\theta + 3 - 3\sin^2\theta = 4$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$

$\sin\theta = \sin 30^\circ$

$\theta = 30^\circ$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

3. $\mathbf{\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}}$

divide numerator & denominator by $\cos\theta$

$$= \frac{3\frac{\sin\theta}{\cos\theta} + \frac{2\cos\theta}{\cos\theta}}{\frac{3\sin\theta}{\cos\theta} - \frac{2\cos\theta}{\cos\theta}}$$

$\left[\frac{\sin\theta}{\cos\theta} = \tan\theta\right]$

$$= \frac{3\tan\theta + 2}{3\tan\theta - 2}$$

put value of $\tan\theta$

$$= \frac{3\times\frac{4}{3}+2}{3\times\frac{4}{3}-2} = \frac{6}{2} = 3$$

4. $\mathbf{\frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ}}$

$$= \frac{\tan 60^\circ - \tan 15^\circ}{\tan 15^\circ - \tan 60^\circ}$$

$$= \frac{-(\tan 15^\circ - \tan 60^\circ)}{\tan 15^\circ - \tan 60^\circ} = -1$$

5. $\mathbf{(2\cos^2\theta - 1)\left[\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}\right]}$

Shortcut Method :

Put $\theta - 0^\circ$

$$(2\times 1 - 1)\left[\frac{1+0}{1-0} + \frac{1-0}{1+0}\right]$$

$1\times 2 = 2$

6. $\mathbf{\cos 1^\circ . \cos 2^\circ . \cos 3^\circ \ldots \cos 100^\circ}$

$\Rightarrow \quad \cos 1^\circ \ldots \underset{\times 0}{\underset{\downarrow}{\cos 90^\circ}} \ldots \cos 100^\circ$

$\Rightarrow \quad = 0$

7. $\mathbf{r\sin\theta = 1}$

$r\cos\theta = \sqrt{3}$

$\Rightarrow \quad \frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3}\tan\theta = 1$

(Add 1 both sides)

$\Rightarrow \quad \sqrt{3}\tan\theta + 1 = 1 + 1$

$\Rightarrow \quad = 2$

8. $\mathbf{x = \frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$

$\Rightarrow \quad \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta} = \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$

$\Rightarrow \quad \frac{\cos\theta}{1+\sin\theta} = \frac{1}{x}$

9. $\mathbf{\tan\theta = \frac{1}{\sqrt{11}} \quad \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}}$

$$= \frac{\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}}$$

$$\Rightarrow \quad = \frac{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta.\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta.\cos^2\theta}}$$

$\Rightarrow \quad = \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$\Rightarrow \quad = \dfrac{\cos^2\theta\left(1-\dfrac{\sin^2\theta}{\cos^2\theta}\right)}{\cos^2\theta\left(1+\dfrac{\sin^2\theta}{\cos^2\theta}\right)}$

$\Rightarrow \quad \dfrac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \dfrac{1-\dfrac{1}{11}}{1+\dfrac{1}{11}}$

$= \dfrac{5}{6}$

10. According to the question

$\tan A = n\tan B$ and $\sin A = m\sin B$

$n = \dfrac{\tan A}{\tan B} m = \dfrac{\sin A}{\sin B}$

Put $A = 30^\circ$

and $B = 60^\circ$

$n = \dfrac{\dfrac{1}{\sqrt{3}}}{\sqrt{3}} m = \dfrac{\dfrac{1}{2}}{\dfrac{\sqrt{3}}{2}}$

$n = \dfrac{1}{3} m = \dfrac{1}{\sqrt{3}}$

$\therefore \cos^2 A = \cos^2 30^\circ = \dfrac{3}{4}$

Now check from option to save your valuable time

11. $\tan 4^\circ \tan 43^\circ \tan 47^\circ \tan 86^\circ$

Here,

$\tan 86^\circ = \tan(90^\circ - 4^\circ) = \cot 4^\circ$

$\tan 47^\circ = \tan(90^\circ - 43^\circ) = \cot 43^\circ$

$\tan 4^\circ.\cot 4^\circ.\tan 43^\circ.\cot 43^\circ = 1$

12. $\cot\theta.\tan(90^\circ - \theta) - \sec(90^\circ - \theta)$

$\operatorname{cosec}\theta + (\sin^2 25^\circ + \sin^2 65^\circ) + \sqrt{3}(\tan 5^\circ$

$.\tan 15^\circ.\tan 30^\circ.\tan 75^\circ \tan 85^\circ)$

$= \cot\theta.\cot\theta - \operatorname{cosec}\theta.\operatorname{cosec}\theta +$

$(\sin^2 25^\circ + \cos^2 25^\circ) + \sqrt{3}(\tan 5^\circ.$

$\tan 85^\circ).(\tan 15^\circ.\tan^2 75^\circ).\tan 30^\circ]$

$= (\cot^2\theta - \operatorname{cosec}^2\theta) + (1) + \sqrt{3}(1.1\dfrac{1}{\sqrt{3}})$

$\left[\begin{array}{c}\tan A.\tan B = 1 \\ \hline If A + B = 90^\circ\end{array}\right]$

$= (-1) + (9) + \sqrt{3} \times \dfrac{1}{\sqrt{3}} = -1 + 1 + 1$

$= 1$

13. $\tan 7\theta.\tan 2\theta = 1$

(If, $\tan A.\tan B = 1$ then, $A + B = 90^\circ$)

$7\theta + 2\theta = 90^\circ$

$9\theta = 90^\circ$

$\theta = 10^\circ$

$\Rightarrow \quad \tan 3\theta$

$\Rightarrow \quad \tan 30^\circ$

$\Rightarrow \quad = \dfrac{1}{\sqrt{3}}$

14. $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ$

$\Rightarrow$

$(\tan 10^\circ \tan 80^\circ)(\tan 15^\circ \tan 75^\circ)$

$\Rightarrow 1 \times 1$

$\Rightarrow (\tan A.\tan B = 1, if A + B = 90^\circ$

$\Rightarrow \quad = 1$

15. $\tan^2\theta.\tan^3\theta = 1$

$2\theta + 3\theta = 90^\circ$

$5\theta = 90^\circ$

(It $\tan A.\tan B = 1$ then $A + B = 90^\circ$)

$\theta = 18^\circ$

16. Given

$\Rightarrow$

$\sin^2 22^\circ + \sin^2 68^\circ + \cot^2 30^\circ$

$\Rightarrow \cos^2 68^\circ + \sin^2 68^\circ + \cot^2 30^\circ$

$\left[\begin{array}{c}\sin(90-\theta) = \cos\theta \\ \hline \cos(90-\theta) = \sin\theta\end{array}\right]$

$\Rightarrow \quad 1 + \cot^2 30^\circ$

$\Rightarrow \quad 1 + (\sqrt{3})$

$(\because \cot 30^\circ = \sqrt{3}$

$\Rightarrow \quad = 4$

17. $\cos 20^\circ = m$

$\cos 70^\circ = n$

So, $m^2 + n^2 = \cos^2 20^\circ + \cos^2 70^\circ$

$\left[\begin{array}{c}\cos^2 A + \cos^2 B = 1 \\ \hline If A + B = 90^\circ\end{array}\right]$

18. $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 180^\circ$

$\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 180^\circ$

$= 0$ (Because $\cos 90^\circ = 0$

Then all terms become '0')

19. $\dfrac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = 2$

By componendo and dividendo,

$\dfrac{2\tan\theta}{2\cot\theta} = \dfrac{3}{1}$

$\dfrac{\sin\theta}{\cos\theta} \times \dfrac{\sin\theta}{\cos\theta} = 3$

$\sin^2\theta = 3\cos^2\theta$

$\sin^2\theta = 3(1 - \sin^2\theta)$

$$4\sin^2\theta = 3$$
$$\sin^2\theta = \frac{3}{4}$$
$$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$$

20. Compo-nado divido

$$\frac{a+b}{a-b} = \frac{m}{n}$$
$$\frac{a+b}{b-b} = \frac{m}{n}$$
$$\frac{a}{b} = \frac{m+n}{b-m-n}$$
$$\frac{\sin\theta + co\theta}{\sin\theta - \cos\theta} = 3$$
$$\sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$$
$$2\sin\theta = 4\cos\theta$$
$$\Rightarrow \quad \tan\theta = \frac{2}{1} = \frac{P}{B}$$
$$\Rightarrow \quad \sin^4\theta - \cos^4\theta$$
$$\Rightarrow (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$$
$$\Rightarrow \quad \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

21. $\boldsymbol{\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}}$

$$4\sin\theta + 4\cos\theta = 5\sin\theta - 5\cos\theta$$
$$\sin\theta = 9\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = 9$$
$$\tan\theta = 9$$
$$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = \frac{9^2+1}{9^2-1}$$
$$= \frac{82}{80} = \frac{41}{40}$$

22. $\boldsymbol{\tan 15^\circ \cot 75^\circ + \tan 75^\circ \cot}$

$= \tan^2 15^\circ . \cot(90^\circ - 15^\circ) + \tan$

$(90^\circ - 15^\circ) . \cot 15^\circ$

$= \tan^2 15^\circ + \cot^2 15^\circ = \tan^2 15^\circ$

$+ \cot^2 15^\circ$ ...(i)

23. $\boldsymbol{\frac{A}{B} = \frac{\tan 11^\circ \tan 29^\circ}{2\cot 61^\circ \cot 79^\circ}}$

$$\frac{A}{B} = \frac{\tan 11^\circ \tan 29^\circ}{2(\cot(90^\circ - 29^\circ)\cot(90^\circ - 11^\circ)}$$
$$\frac{A}{B} = \frac{\tan 11^\circ \tan 29^\circ}{2\tan 11^\circ \tan 29^\circ}$$
$$\frac{A}{B} = \frac{1}{2}$$
$$2A = B$$

24. $\boldsymbol{2(\cos^2\theta - \sin^2\theta) = 1}$

$$\Rightarrow \quad \cos^2\theta - \sin^2\theta = \frac{1}{2}$$
$$\Rightarrow \quad 2\sin^2\theta = 1 - \frac{1}{2}$$
$$\Rightarrow \quad \sin\theta = \frac{1}{4}$$
$$\Rightarrow \quad \sin\theta = \frac{1}{2} = \sin 30^\circ$$
$$\theta = 30^\circ$$

25. $\boldsymbol{\tan(2\theta + 45^\circ) = \cot 3\theta}$

(*if* $\tan A = \cot B$ then $A + B = 90^\circ$

$$(2\theta + 45^\circ + 3\theta = 90^\circ$$
$$5\theta + 45^\circ = 90^\circ$$
$$\theta = \frac{45}{5} = 9^\circ$$

26. $\boldsymbol{2\sin\left(\frac{nx}{2}\right) = x^2 + \frac{1}{x^2}}$

Let $x = 1$

$$2\sin 90. = 1^1 + \frac{1}{1^2}$$
$$2 \times 1 = 1 + 1$$
$$2 = 2 \text{ matched, so } x = 1$$

so, $x - \dfrac{1}{x}$

$$1 - \frac{1}{1} = 0$$

27. $\boldsymbol{x\sin 45^\circ - y\,\mathrm{cosec}\,30^\circ}$

$$\frac{x}{y} = \frac{\mathrm{cosec}\,30^\circ}{\sin 45^\circ} = \frac{2}{1/\sqrt{2}} = \frac{2\sqrt{2}}{1}$$
$$\frac{x^4}{y^4} = \left(\frac{2\sqrt{2}}{1}\right)^4 = \frac{64}{1} = 4^3$$

28. $\boldsymbol{\theta = 60^\circ}$

$$\frac{1}{2}\sqrt{1+\sin\theta} + \frac{1}{2}\sqrt{1-\sin\theta}$$
$$\frac{1}{2}\sqrt{1+\sin 60^\circ} + \frac{1}{2}\sqrt{1-\sin 60^\circ}$$
$$\frac{1}{2}\sqrt{1+\left(\frac{\sqrt{3}}{2}\right)} + \frac{1}{2}\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)}$$
$$\frac{1}{2\sqrt{2}}(\sqrt{2+3} + \sqrt{2-\sqrt{3}})$$
$$\frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}}(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})$$
$$\frac{1}{4}(\sqrt{(\sqrt{3}+1)^2} + \sqrt{(\sqrt{3}-1)^2})$$
$$\frac{1}{4}(\sqrt{3} + 1 + \sqrt{3} - 1)$$
$$2\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} = \cos 30^\circ = \cos\frac{\theta}{2}$$

29. $\boldsymbol{x\sin 60^\circ \tan 30^\circ - \tan^2 45^\circ = \mathrm{cosec}\,60^\circ \cot 30^\circ}$

$-\sec^2 45^\circ$

$$x \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} - 1$$

$$= \frac{2}{\sqrt{3}} \times \sqrt{3} - (\sqrt{2})^2$$

$$\frac{x}{2} - 1 = 2 - 2$$

$$\frac{x}{2} - 1 = 0$$

$$\frac{x}{2} = 1 = x = 2$$

30. $1 + \frac{1}{\cot^2 63^\circ} - \sec^2 27^\circ + \frac{1}{\sin^2 63^\circ} - \operatorname{cosec}^2 27^\circ$

$1 + \tan^2 63^\circ - \sec^2 27^\circ + \operatorname{cosec}^2 63^\circ - \operatorname{cosec}^2 27^\circ$

$1 + \cot^2 27^\circ - \sec^2 27^\circ + \sec^2 27^\circ - \operatorname{cosec}^2 27^\circ$

$1 + \cot^2 27^\circ - \operatorname{cosec}^2 27^\circ$

$= 1 - 1 = 0$

31. $\sin(4\alpha - \beta) = 1 = \sin 90^\circ$

$$\cos(2\alpha + \beta) = \frac{1}{2} = \cos 60^\circ$$

$$4\alpha - \beta = 90^\circ$$

$$2\alpha + \beta = 60^\circ$$

adding $\quad 6\alpha = 150^\circ$

$$\alpha = 25^\circ$$

$$\beta = 10^\circ$$

$$\sin(\alpha + 2\beta)$$

$$\sin(25^\circ + 2 \times 10^\circ)$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

32. $4\cos^2\theta - 1 = 0$

$$4\cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{2} = \cos 60^\circ$$

$$\theta = 60^\circ$$

$$\tan(\theta - 15^\circ)$$

$$\tan(60^\circ - 15^\circ) = \tan 45^\circ = 1$$

33. According to the question,

$$\sec x + \cos x = 2$$

Put $\quad x = 0$

$$\sec 0^\circ + \cos 0^\circ = 2$$

$$1 + 1 = 2$$

$$2 = 2 \text{ (Satisfy)}$$

$\therefore \quad \sec^{16} x + \cos^{16} x$

$$\sec^{16} 0 + \cos^{16} 0$$

$$\sec 0^\circ + \cos 0^\circ$$

$$1 + 1 = 2$$

34. Shortcut Method

Put $\quad \theta = 30^\circ$

$$\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$$

$$\sec 30^\circ - \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ (satisfy)}$$

$$\sec\theta = 30^\circ$$

$$\sec\theta . \tan\theta$$

$$\sec 30^\circ . \tan 30^\circ$$

$$\frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2}{3}$$

35. $\sin(\theta + 30^\circ) = \frac{3}{\sqrt{12}} = \frac{3}{2\sqrt{3}} - \frac{\sqrt{3}}{2}$

$$\sin(\theta + 30^\circ) = \sin 60^\circ$$

$\therefore \quad \theta = 30^\circ$

$$\cos^2\theta = \cos^2 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{3}{4}$$

36. $\sin\theta + \operatorname{cosec}\theta = 2$

put value $\theta = 90^\circ$

$\sin 90^\circ + \operatorname{cosec} 90^\circ = 2$

$1 + 1 = 2$

$2 = 2$

It, satisfies the question $\sin^5\theta + \operatorname{cosec}^5\theta$

$= \sin^5 90^\circ + \operatorname{cosec}^5 90^\circ$

$= (1)^5 + (1)^5$

$=$ १ + १ = २

37. $\sec^2\theta + \tan^2\theta = 7$

$1 + \tan^2\theta + \tan^2\theta = 7$

$$2\tan^2\theta = 6$$

$$\tan^2\theta = 3$$

$$\tan\theta = \sqrt{3}$$

$$\theta = 60^\circ$$

Alternate :

take help from option

put $\theta = 60^\circ$

$$\sec^2 60^\circ + \tan^2 60^\circ = 7$$

$$(2)^2 + (\sqrt{3})^2 = 7$$

$$7 = 7 \text{ (matched)}$$

So $\quad \theta = 60^\circ$

38. $11^\circ 15' = 11 + \frac{15}{60} = 11 + \frac{1}{4} = \frac{45^\circ}{4}$

We know π radian $= 180^\circ$

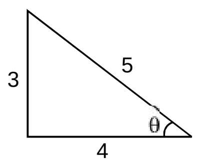
$$1^\circ = \left(\frac{\pi}{180}\right) \text{radian},$$

$$\frac{45^\circ}{4} = \frac{\pi}{180^\circ} \times \frac{45^\circ}{4} = \frac{\pi^c}{16}$$

39. $\cos\theta = \dfrac{\mathbf{15 \to Base}}{\mathbf{17 \to Hypo.}}$

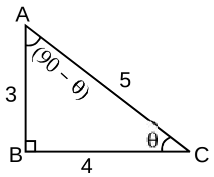
40. Shortcut method

$$\tan\theta = \frac{3}{4} = \tan\theta = \frac{\text{'}P\text{'}}{\text{'}B\text{'}}$$



$$\text{cosec}\,\theta = \frac{H}{P} = \frac{5}{3}$$

41. $\sin\theta = \dfrac{\mathbf{3}}{\mathbf{5}}$



$$\sin\theta = \frac{3}{5} = \frac{P}{H}$$

So, $B = 4$

$P = 3$

$H = 5$

$$\frac{\tan\theta + \cos\theta}{\cot\theta + \text{cosec}\,\theta} = \frac{\frac{P}{B} + \frac{B}{H}}{\frac{B}{P} + \frac{H}{P}}$$

$$\frac{\frac{3}{4} + \frac{4}{5}}{\frac{4}{3} + \frac{5}{3}} = \frac{\frac{15+16}{20}}{\frac{4+5}{3}} = \frac{\frac{31}{20}}{\frac{9}{3}} = \frac{31}{60}$$

42. Let the length of string $AC = l$ metre

Height $AB = 75$ m.

$\because$ $\cot\theta = \dfrac{Base}{Perpendicular}\dfrac{8}{15}$

Then Hypotenuse $\to$ 17

Using triplet 8, 15, 17

15 units $\to$ 75 m

1 unit $\to$ 5

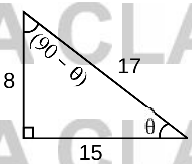
Length of chord $AC = l = 17$ units

$= 17 \times 5$

$= 85$ metres.

43. $\sec\theta + \tan\theta = \sqrt{3}$ **...(i)**

$$\sec^2\theta - \tan^2\theta = 1$$



Perpendicular = 8

$$= \cot(90^\circ - \theta) = \tan\theta = \frac{8}{15} \quad \left[\therefore \tan\theta = \frac{P}{B}\right]$$

$$[1 + \tan^2\theta = \sec^2\theta]$$

$$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$$

$$\sec\theta - \tan\theta = \frac{1}{\sqrt{3}} \quad \text{...(ii)}$$

subtraction equation (i) from (ii)

$$2\tan\theta = \sqrt{3} - \frac{1}{\sqrt{3}}$$

$$2\tan\theta = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\theta = 30^\circ \left[\tan 30^\circ = \frac{1}{\sqrt{3}}\right]$$

$$\tan 3\theta = \tan 90^\circ \text{ (undefined)}$$

44. $\sec\theta + \tan\theta = 2 + \sqrt{5}$ **...(i)**

$$\sec^2\theta - \tan^2\theta = 1$$

$$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$$

$$(\sec\theta - \tan\theta) = \frac{1}{2+\sqrt{5}} = \frac{1}{\sqrt{5}+2}$$

$$= \sqrt{5} - 2 \quad \text{...(ii)}$$

ad eq (i) + (ii)

$$2\sec\theta = 2 + \sqrt{5} + \sqrt{5} - 2$$

$$2\sec\theta = 2\sqrt{5}$$

$$\sec\theta = \sqrt{5}$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \left(\frac{1}{\sqrt{5}}\right)^2$$

$$\sin^2\theta = \frac{4}{5}$$

$$\sin\theta = \frac{2}{\sqrt{5}}$$

$$\therefore \quad \sin\theta + \cos\theta = \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{3}{\sqrt{5}}$$

45. Given

$$\sec\theta + \tan\theta = p \quad \text{...(i)}$$

then $$\sec\theta - \tan\theta = \frac{1}{p} \quad \text{...(ii)}$$

From equation (i) + (ii)

$$2\sec\theta = p + \frac{1}{p}$$

$$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$$

46. $\mathbf{3(\sec^2\theta + \tan^2\theta) = 5}$

$$\sec^2\theta + \tan^2\theta = \frac{5}{3} \quad \ldots(i)$$

$$\sec^2\theta - \tan^2\theta = 1 \quad \ldots(ii)$$

Add eqn (i) & (ii)

$$2\sec^2\theta = \frac{8}{3}$$

$$\sec\theta = \frac{2}{\sqrt{3}}$$

$$\therefore \quad \theta = 30^\circ$$

$$\cos 2\theta = \cos 2(30^\circ) = \cos 60^\circ = \frac{1}{2}$$

47. $\mathbf{\frac{5\pi}{9} = \frac{5 \times 180^\circ}{9} = 100^\circ}$

Other two angle must be 40°+40°

$$40^\circ = 40^\circ = \frac{2\pi}{9}$$

48. $\angle \boldsymbol{A} + \angle \boldsymbol{B} = \mathbf{135^\circ}$ **...(i)**

$$\angle A - \angle B = \frac{\pi}{12} = 15^\circ \quad \ldots(ii)$$

adding both equation

$$2\angle A = 150^\circ$$

$$\angle A = 75^\circ$$

$$\angle B = 60^\circ$$

49. $\alpha + \theta = \mathbf{\frac{7\pi}{12}}$ **...(i)**

$$\tan\theta = \sqrt{3}$$

$$\tan\theta = \tan 60^\circ$$

$$\theta = 60^\circ$$

Put value in equation (i)

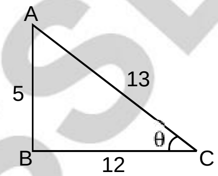
$$\alpha + 60^\circ = \frac{7}{12} \times 180^\circ$$

$$\alpha = 105^\circ - 60^\circ$$

$$\alpha = 45^\circ$$

$$\tan\alpha = \tan 45^\circ = 1$$

50. $\mathbf{\sin\theta = \frac{5}{13}}$



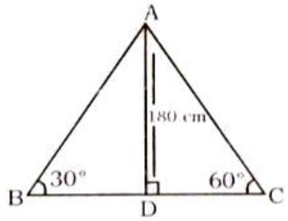
$$BC = \sqrt{169 - 25} = 12$$

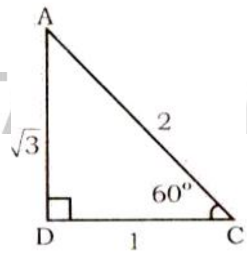
$$\sqrt{\cot\theta + \tan\theta} = \sqrt{\frac{12}{5} + \frac{5}{12}}$$

$$= \sqrt{\frac{169}{60}} = \frac{13}{2\sqrt{15}}$$

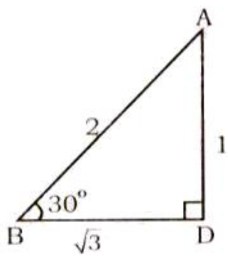
HEIGHT AND DISTANCE SOLUTION

1.





$$AD:CD = \sqrt{3}:1$$



$$AD:BD = 1:\sqrt{3}$$

From Eq. (i) & (ii) to make equal ratio

$$CD:AD:BD = 1:\sqrt{3}:3$$

$$BC = DC + BD$$

$$BC = 1 + 3 = 4\text{ units}$$

$\sqrt{3}$ units $\rightarrow$ 180 m

1 unit $= \dfrac{180}{\sqrt{3}}$

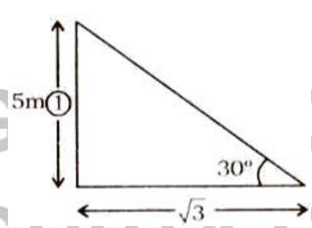
Now, Distance between the two ships

4 units $= \dfrac{180}{\sqrt{3}} \times 4$

$$= \frac{180 \times 4 \times \sqrt{3}}{3} = 240\sqrt{3}$$

$$= 415.68$$

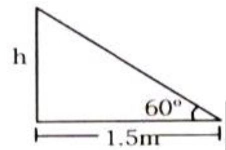
2.



1 unit $= 5$ m

$\sqrt{3}$ units $= 5\sqrt{3}$ m

3. Let height of pole = *h* mtr



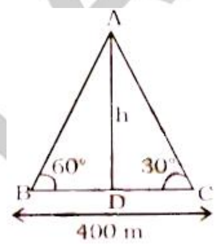
$$\tan 60^\circ = \frac{h}{1.5}$$

$$h = 1.5 \times \tan 60^\circ$$

$$= 1.5 \times \sqrt{3}$$

$$= \frac{3\sqrt{3}}{2}\text{ mr}$$

4.



$$BC = 400\text{ metres}$$

In ΔABD

$$\tan 60^\circ = \frac{AD}{DC}$$

$$\frac{1}{\sqrt{3}} = \frac{AD}{DC}$$

$\Rightarrow$ $AD:DC = 1:\sqrt{3}$ ...(ii)

Now,

$$BC = BD + DC$$

$$= 1 + 3 = 4\text{units}$$

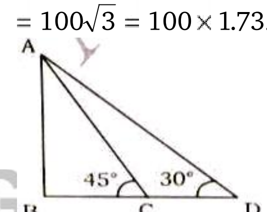
$$4\text{ units} = 400\text{ m}$$

$$1\text{ unit} = 100\text{ m}$$

$$AD = \sqrt{3}\text{ units}$$

$= 100\sqrt{3} = 100 \times 1.732 = 173.2$ m

5.



AB= height of peak $= 300$ m

CD = length of Bridge

In ΔABC

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC} = AB:BC = 1:1$$

In ΔABD $\quad \tan 30^\circ = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD}$$

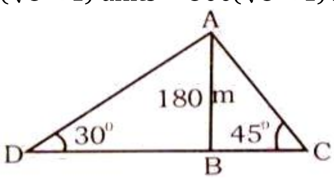
$\Rightarrow$ $\quad AB:BD = 1:\sqrt{3}$

$$CD \Rightarrow \sqrt{3}-1$$

$$AB = 1 = 300\text{metre}$$

$(\sqrt{3}-1)$ units $= 300(\sqrt{3}-1)$ metre

6.



$$AB = 180\text{ m}$$

$$CD = 1$$

In ΔABC

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC} \Rightarrow AB:BC = 1:1 \ldots(1)$$

In ΔABD $\quad \tan 30^\circ = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD}$$

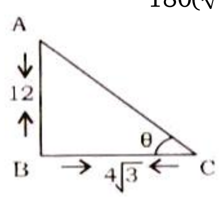
$$AB:BD = 1:\sqrt{3} \ \ldots\text{(ii)}$$

$$= (\sqrt{3}+1)\text{ units}$$

$$AB = 1\text{units} \quad = 180\text{ unit}$$

$$180(\sqrt{3}+1)\text{m}$$

7.



In ΔABC

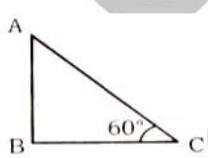
$$\tan\theta = \frac{AB}{BC} = \frac{12}{4\sqrt{3}}$$

$$\tan\theta = \frac{3}{\sqrt{3}}$$

$$\tan\theta = \sqrt{3} = \tan 60^\circ$$

$$\theta = 60^\circ$$

8.



$AC =$ Ladder

$BC = 6.5$ metres

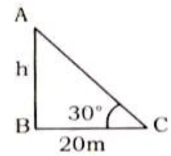
In ΔABC

$$\cos 60^\circ = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{6.5}{AC}\text{ m}$$

$$AC = 13\text{ m}$$

9.



$AC =$ Ladder

$BC = 6.5$ metres

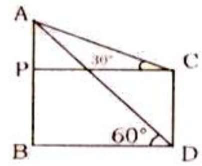
In ΔABC

$$\cos 60^\circ = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{6.5}{AC}\text{ m}$$

$$AC = 13\text{ m}$$

10.



$AB =$ hill $= 200$ metre

$CD =$ tower

In ΔAPC $\quad \tan 30^\circ = \frac{AP}{PC}$

$$\frac{1}{\sqrt{3}} = \frac{AP}{PC} = AP:PC = \sqrt{3}:1$$

In ΔABD $\quad \tan 60^\circ = \frac{AB}{BD}$

$$\sqrt{3} = \frac{AB}{BD} = AB:BD = \sqrt{3}:1 \ldots\text{(ii)}$$

$$PB = CD \text{ and } PC = BD$$

Now $\quad CD = PB \Rightarrow AB - AP$

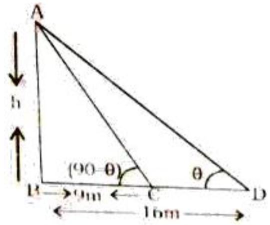
$$CD = 3-1 = 2\text{ units}$$

$$AB = 3\text{ units} = 200\text{ units}$$

$$CD = 2\text{ units} = \frac{200}{3}\times 2$$

$$= 133\frac{1}{3}\text{ metre}$$

11.



$AB =$ Pillar

$BC = 9$ metre

$BD = 16$ metre

$\angle ADB = Q$

In ΔABC

$$\tan(90-\theta) = \frac{AB}{BC}$$

$$\cot\theta = \frac{AB}{BC} = \frac{h}{9} \quad ...(i)$$

In ΔABD

$$\tan\theta = \frac{h}{16} \quad ...(ii)$$

By multiplying equaltion (i) and (ii)

$$\tan.\cot\theta = \frac{h}{9}\times\frac{h}{16}$$

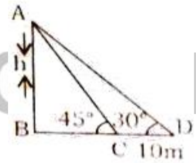
$\Rightarrow$ $$\frac{h^2}{144} = 1$$

$\Rightarrow$ $$h^2 = 144$$

$$h = \sqrt{144}$$

$$h = 12 \text{ metre}$$

12.



AB= tower= h

In ΔABC

$$\tan 45^\circ = \frac{AB}{BC} = 1$$

$$AB:BC = 1:1 \quad ...(i)$$

In ΔABD

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD} = AB:BD = 1:\sqrt{3} \quad ...(ii)$$

Now,

$$CD = BD - BC$$

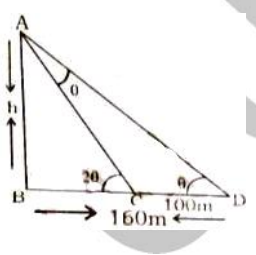
$$= \sqrt{3}-1$$

$$(\sqrt{3}-1)\text{units} = 10\text{ m}$$

$(AB) = 1$ units $$= \frac{10}{\sqrt{3}-1}$$

$$= 5(\sqrt{3}+1) \text{ metre}$$

13.



$$BD = 160\text{ m}$$

In ΔACD

exter. $$\angle ACB = \angle CAD + \angle ADC$$

$$2\theta = \angle CAD + \theta$$

$\therefore$ $$AC = CD$$

$$AC = 100\text{ m}$$

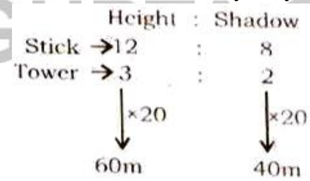
In ΔABC

$$AC = 100\text{ m}$$

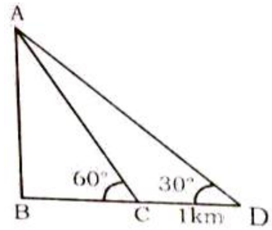
$$BC = 160 - 100 = 60\text{ m}$$

Then $AB = 80$ m (by Phytagores theorem)

14.



15. It should be on circumcentre.



AB = height of balloon

In ΔABC

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{BC} \Rightarrow AB:BC = \sqrt{3}:1$$

In ΔABD

$$\tan 30^\circ = \frac{AB}{BD} \Rightarrow AB:BD = 1:\sqrt{3}$$

Now,

$$CD = BD - BC$$

$$= 3 - 1 = 2\text{ units}$$

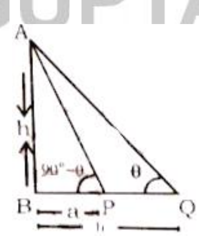
$$2 \text{ units} = 1\text{ km}$$

$$1 \text{ unit} = \frac{1}{2}$$

$$AB = \sqrt{3}\text{unit} = \frac{1}{2}\times\sqrt{3}$$

$$= \frac{\sqrt{3}}{2}\text{ km}$$

16.



AB is tower

$\therefore$ $$\angle AQB = \theta$$

$\therefore$ $$\angle APB = 90^\circ - \theta$$

$$PB = a, BQ = b$$

In ΔAQB

$$\tan\theta = \frac{AB}{BQ}$$

$$\tan\theta = \frac{h}{b} \quad ...(i)$$

In ΔAPB

$$\tan(90^\circ-\theta)=\frac{h}{PB}$$

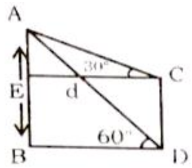
$\Rightarrow$ $$\cot\theta=\frac{h}{a}\quad \ldots\text{(ii)}$$

By multiplying both equaltion

$$\tan\theta.\cot\theta=\frac{h}{b}\times\frac{h}{a}$$

$$h^2=ab\Rightarrow h=\sqrt{ab}$$

17.



AB and CD are temples

$BD=$ width of river

$$AB=54\text{m}$$

In ΔAEC

$$\tan 30^\circ=\frac{AE}{EC}=\frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$AE:EC=1:\sqrt{3}$$

In ΔABD

$$\tan 60^\circ=\frac{AB}{BD}$$

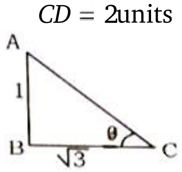
$$\sqrt{3}=\frac{AB}{BD}\Rightarrow AB:BD=\sqrt{3}:1\ \ldots\text{(ii)}$$

$EB=CD$ and $EC=BD$

Now,

$$CD=AB-AE$$
$$=3-1=2\text{ units}$$
$$AB=3\text{ units}\times 18-54\text{ m}$$
$$CD=2\text{units}\times 18=36\text{m}$$

18.



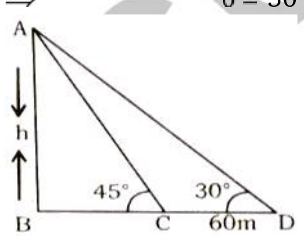
In ΔABC

$$\tan\theta=\frac{1}{\sqrt{3}}$$

$$\tan\theta=\tan 30^\circ$$

$\Rightarrow$ $$\theta=30^\circ$$

19.



$h=$height

In ΔABC

$$\tan 45^\circ=\frac{AB}{BC}$$

$$\frac{1}{1}=\frac{AB}{BC}=AB:BC=1:1\quad\ldots\text{(i)}$$

In ΔABD

$$\tan 30^\circ=\frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}}=\frac{AB}{BD}\quad\Rightarrow$$

$$AB:BD=1:\sqrt{3}\ \ldots\text{(ii)}$$

Now,

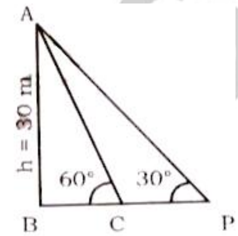
$$CD=BD-BC$$
$$CD=\sqrt{3}-1$$
$$\sqrt{3}-1\text{units}=60$$

$H=1$ unit $$=\frac{60}{\sqrt{3}-1}$$

$$=\frac{60}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$h\Rightarrow 30(\sqrt{3}+1)\text{ m}$$

20.



$$h=30^\circ$$
$$PC=?$$

In ΔABP

$$\tan 30^\circ=\frac{AB}{BP}$$

$$\frac{1}{\sqrt{3}}=\frac{AB}{BP}\Rightarrow AB:BP=1:\sqrt{3}\ldots\text{(i)}$$

In ΔABC

$$\tan 60^\circ=\frac{AB}{BC}$$

$$\frac{\sqrt{3}}{1}=\frac{AB}{BC}\Rightarrow AB:BC=\sqrt{3}:1\ ..\text{(ii}$$

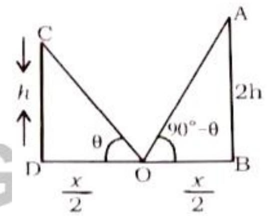
Now,

$$AB=\sqrt{3}\text{units}=30\text{ metre}$$

1 unit $$=\frac{30}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=10\sqrt{3}$$

$$PC=3-1=2\text{ units}$$
$$=10\sqrt{3}\times 2=20\sqrt{3}\text{ metre}$$

21.



$$OB=OD=\frac{x}{2}$$

In ΔOCD

$$\tan\theta = \frac{h}{\frac{x}{2}} \Rightarrow \frac{2h}{x} \ldots(i)$$

In ΔAOB

$$\tan(90^\circ - \theta) = \frac{AB}{30}$$

$$\Rightarrow \qquad \cot\theta = \frac{\frac{2h}{x}}{2} = \frac{4h}{x} \ldots(ii)$$

Multiplying both equations

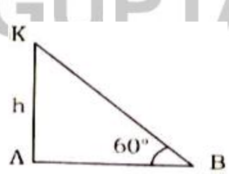
$$\tan\theta.\cot\theta = \frac{2h}{x} \times \frac{4h}{x}$$

$$\Rightarrow \qquad x^2 = 8h^2$$

$$\Rightarrow \qquad h^2 = \frac{x^2}{8}$$

$$\Rightarrow \qquad h = \frac{x}{2\sqrt{2}} \text{ metre}$$

22.



K =kite

KB = thread = 150 metre

KA = height of kite from ground

In ΔKAB

$$\tan 60^\circ = \frac{KA}{AB}$$

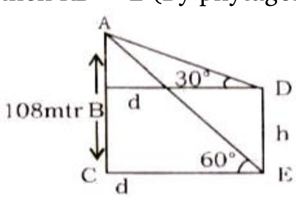
$$\frac{\sqrt{3}}{1} = \frac{KA}{AB}$$

If h $= \sqrt{3}$

$AB = 1$

then KB = 2 (By phytagoras theorem

23.



In ΔACE

$$\tan 60^\circ = \frac{AC}{CE}$$

$$\frac{\sqrt{3}}{1} = \frac{AC}{CE} = AC:CE = \sqrt{3}:1 \ldots(i)$$

In ΔABD

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow \sqrt{3}h = x \ldots(i)$$

In ΔCDP

$$\tan 60^\circ = \frac{h}{(100-x)}$$

$$\Rightarrow \qquad \sqrt{3}(100-x) = h$$

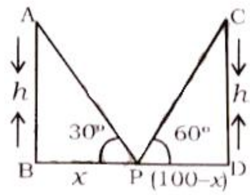
$$\Rightarrow \qquad \sqrt{3}(100-\sqrt{3}h) = h$$

(Put the value of x from equation (i)

$$\Rightarrow \qquad 100\sqrt{3} - 3h \Rightarrow 4h = 100\sqrt{3}$$

$$h = 25\sqrt{3} \text{ metre}$$

24.



$BD = 100$

$AB = CD = h$' metre (Height of pole in ΔABP

$$\Rightarrow \qquad \tan 30^\circ = \frac{h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow \sqrt{3}h = x \ldots(i)$$

In ΔCDP

$$\tan 60^\circ = \frac{h}{(100-x)}$$

$$\Rightarrow \qquad \sqrt{3}(100-x) = h$$

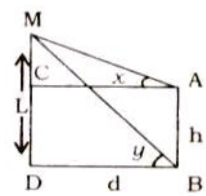
$$\Rightarrow \qquad \sqrt{3}(100-\sqrt{3}h) = h$$

Put the value of x from equation (i)

$$\Rightarrow \qquad 100\sqrt{3} - 3h = h \Rightarrow 4h = 100\sqrt{3}$$

$$h = 25\sqrt{3} \text{ metre}$$

25.



AB= tree 'h'

MD = Building 'I'

$DB = CA$ =' d'

In ΔMCA

$$\tan x = \frac{MC}{AC} = \frac{1-h}{d}$$

$$\Rightarrow d = \frac{l-h}{\tan x} \Rightarrow d = (1-h)\cot x$$

In ΔMDB

$$\text{tany} \qquad = \frac{1}{d} = \frac{MD}{DB}$$

from equation (i) and (ii)

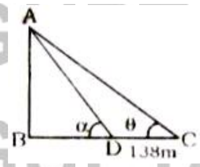
$$(1-H)\cot x = l\cot y$$

$$(1-h)\cot x = l\cot y$$

$$l\cot x - h\cot x = l\cot$$

$$h\cot x = l(\cot x - \cot y)$$

$$l = \frac{h\cot x}{\cot x - \cot y}$$

26. 

Shortcut approach

Ist case:

$$\tan\theta = \frac{AB}{BC} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{5}$$

II case:

$$\sec\alpha = \frac{AD}{BD} = \frac{\text{Hypo}}{\text{Base}}$$

$$= \frac{\sqrt{193}}{12}$$

In ΔABD

$$\text{Hypo} = \sqrt{193}$$

$$\text{Base} = 12$$

Then perpendicular $= 7$

(By pythagores theorem

In case I perpendicular is 1

Test Your Skill

COORDINATE SOLUTION

1.B	2.C	3.B	4.C	5.A	6.A	7.B	8.A
9.A	10.C	11.A	12.B	13.C	14.C	15.D	

1. (b)

$$\sqrt{(x-a)^2+(y-0)^2}=a+x$$

$$\Rightarrow (x-a)^2+y^2=(a+x)^2$$

$$\Rightarrow y^2=(x+a)^2-(x-a)^2$$

$$\Rightarrow y^2=4ax$$

2.(c)

माना (x, y), $Q(a+b, b-a)$ तथा $R(a-b, a+b)$ दिए गए बिंदु हैं ।

$\therefore$ PQ = PR.

$$\Rightarrow \sqrt{\{x-(a+b)\}^2+\{y-(b-a)\}^2}$$

$$= \sqrt{\{x-(a-b)\}^2+\{y-(a+b)\}^2}$$

$\Rightarrow x^2-2x(a+b)+(a+b)^2+y^2-2y(b-a)+(b+a)^2 = x^2+(a-b)^2-2x(a-b)+y^2+(a+b)^2-2y(a+b)$

$\Rightarrow ax+bx+by-ay=ax-bx+ay+by$

$\Rightarrow 2bx=2ay$

$\Rightarrow bx=ay$

3. (b)

मध्य बिंदु के निर्देशांक $\equiv (-1, 4)$

$$\therefore \frac{a+a+4}{2}=4$$

$\Rightarrow 2a+4=8$

$\Rightarrow 2a=4$

$a=2$

4. C.

चूंकि, P, Q तथा R सरेखीय हैं ।

$\therefore$ PQ की प्रवणता = PR की प्रवणता

$$\Rightarrow \frac{a-3}{5-2}=\frac{7-3}{6-2}\Rightarrow\frac{a-3}{3}=\frac{4}{4}$$

$\Rightarrow a-3=3 \Rightarrow a=6$

5. (a)

x-अक्ष के समांतर रेखा का समीकरण : $y = b$.
चूंकि यह $(-6, -5)$ से गुजरती है इसलिए $b = -5$

$\therefore$ अभीष्ट समीकरण $y=-5$ है ।

6. (a)

y-अक्ष के समान्तर रेखा का समीकरण : $x=a$.
चूँकि यह $(2, -5)$ से गुजरती है इसलिए $a = 2$
अभीष्ट समीकरण $x=2$ है ।

7. (b)

माना R के निर्देशांक (x, y) हैं तब,

$$\frac{-1+5+x}{3}=4 \text{ and } \frac{0-2+y}{3}=0$$

या $4+x=12$ तथा $-2+y=0$

या $x=8$ तथा $y=2$

$\therefore R=(x, y)=(8, 2)$

8. (a)

माना समीकरण : $y=5x+c$

चूँकि यह $(-4, 1)$ से गुजरती है

We have $1=5(-4)+c$

$c=21$

अत: इसका समीकरण है : $y=5x+21$

9. (a)

10.C.

(c) समान्तर होने की शर्त : $\frac{a_1}{a_2}=\frac{b_1}{b_2}$

$$\therefore \frac{1}{a}=\frac{3}{12}\Rightarrow a=4$$

Alternatively,

$x+3y-8+0$

$$\Rightarrow y=\left(-\frac{1}{3}\right)x+\left(\frac{8}{3}\right)$$

$$\therefore m_1=-\frac{1}{3}$$

$ax+12y+5+0=0$

$$\Rightarrow y=\left(-\frac{a}{12}\right)x-\frac{5}{12}$$

$$\therefore m_2=-\frac{a}{12}$$

समांतर होने के लिए, $m_1=m_2$

$$\therefore -\frac{1}{3}=-\frac{a}{12}\Rightarrow a=4$$

11. (a)

$$2y-\sqrt{12}.x-9=0\Rightarrow y=\frac{\sqrt{12}}{2}x+\frac{9}{2}$$

$$\Rightarrow m_1=\frac{\sqrt{12}}{2}=\sqrt{3}$$

$$\sqrt{3}y-x+7=0\Rightarrow y=\left(\frac{1}{\sqrt{3}}\right)x-\frac{7}{\sqrt{3}}$$

$$\Rightarrow m_2=\frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{m_1-m_2}{1+m_1.m_2}=\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}.\frac{1}{\sqrt{3}}}=\frac{1}{\sqrt{3}}$$

So, $\theta=30°$

12. (b)

PQ की प्रवणता, $m_1=\frac{5-5}{4-3}=0$

PR की प्रवणता, $m_2=\frac{6-5}{4-3}=1$

$$\therefore \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{0-1}{1+0}\right| = 1$$

अत: $\theta = 45°$

13.(c)

(c) स्पष्ट है कि M, QR का मध्य बिंदु है ।

M के निर्देशांक = $\left(\frac{-3-1}{2}, \frac{7-3}{2}\right) = (-2, 2)$

अब बिंदु P(2, 3) तथा M(−2, 2) को मिलाने वाली रेखा का समीकरण :

$$(y-3) = \frac{2-3}{-2-2}(x-2)$$

$$\Rightarrow y-3 = \frac{1}{4}(x-2) \Rightarrow 4y-12 = x-2$$

$$\Rightarrow x-4y+10=0$$

14.C.

दी गई रेखा : $3x+4y-5=0$

$$\Rightarrow y = \left(-\frac{3}{4}\right)x + \frac{5}{4}$$

इसकी प्रवणता = $m_1 = -\frac{3}{4}$

माना m_2 अभीष्ट रेखा की प्रवणता है

तब, $m_1 m_2 = -1$ या $\left(-\frac{3}{4}\right) m_2 = -1$

$$\Rightarrow m_2 = \frac{4}{3}$$

माना अभीष्ट रेखा का समीकरण: $y = m_2 x + c$

$$\Rightarrow y = \frac{4}{3}x + c$$

चूँकि, यह (1, 1) से गुजरती है ।

$$\therefore 1 = \frac{4}{3} \times 1 + c \Rightarrow c = 1 - \frac{4}{3} = -\frac{1}{3}$$

अभीष्ट समीरण : $y = \frac{4}{3}x - \frac{1}{3}$

या $4x-3y-1=0$

15. (d)

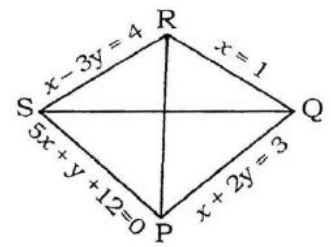
$x+2y=3$ .... (i)

$5x+y=-12$ ....(ii)

समी. (i) तथा (ii) को हल करने पर, हम पाते

$x=-3,\ y=3$

$\therefore$ P के निर्देशांक = (− 3, 3)



इसीप्रकार, Q(1, 1), R(1, −1) तथा S(−2, 2)

अब, m_1 = PR की प्रवणता = $\frac{-1-3}{1+3} = -1$

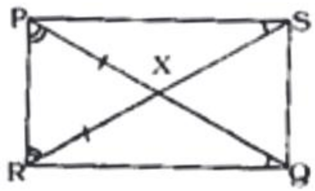
m_2 = QS की प्रवणता = $\frac{-2-1}{-2-1} = 1$

$\because m_1 m_2 = -1$

अभीष्ट कोण = 90°

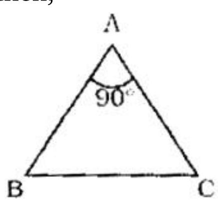
Line & Triangle Sotluions

1.



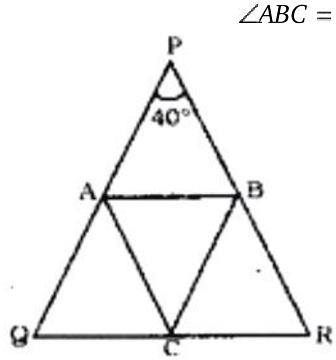
$$XP = XR$$
$$\angle XPR = \angle XRP$$
If $\quad \angle PSX = \angle RQX$
then, $\quad PS = RQ$

2.



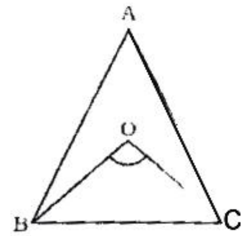
$$AB^2 + AC^2 = BC^2 \Rightarrow \angle BAC = 90^\circ$$
$\Rightarrow \quad AB^2 + AC^2 = 2AB^2$
$\Rightarrow \quad AB^2 = AC^2$
$\Rightarrow \quad AB = AC$
$$\angle ABC = \angle ACB - 45^\circ$$

3.



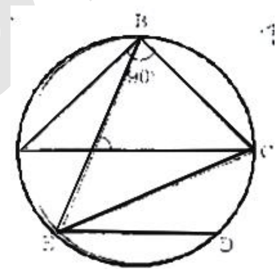
$$AC = QC$$
$\therefore \quad \angle CBR = \angle CRB = y$
$\therefore$ From ΔPQR,
$$\angle x + \angle y + 40^\circ = 180$$
$$\angle x + \angle y = 140^\circ$$
Again,
$$\angle ACQ + \angle ACB + \angle BCR = 180^\circ$$
$\Rightarrow \quad 180^\circ - 2x + \angle ACB + 180^\circ - 2y$
$$= 180^\circ$$
$\Rightarrow \quad \angle ACB = 2(x + y)$
$$2 \times 140 - 180^\circ = 100^\circ$$
$$= 180^\circ$$
$$\angle ACB = 2(x + y)$$

4.



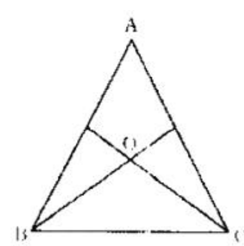
In ΔABC
$$\angle A + \angle B + \angle C = 180^\circ$$
ΔOBC,
$$OBC + \angle BOC + \angle OCB = 180^\circ$$
$$= \frac{\angle B}{2} + 110 + \frac{\angle C}{2} - 180^\circ$$
$$= \frac{\angle B + \angle C}{2}$$
$$= 180^\circ - 110^\circ = 70^\circ$$
$$= \angle B + \angle C = 140^\circ$$
$$\angle A = 180^\circ - 140^\circ = 40^\circ$$

5.



$$\angle BE = 50^\circ$$
$$\angle BAC + \angle BCA = 90^\circ$$
$$\angle BE = 90^\circ - 50^\circ = 40^\circ$$
$$\angle ABE = \angle ACE = 40^\circ$$
$$\angle ACE = \angle DEC = 40^\circ$$

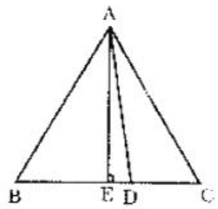
6.



ΔABC,
$$\Delta + \angle B + \angle C = 180^\circ$$
$$\Delta BOC, \angle BOC = 110^\circ$$
$$\frac{B}{2} + \frac{C}{2} = 180^\circ - 110^\circ = 70^\circ$$
$$B.C = 140^\circ$$

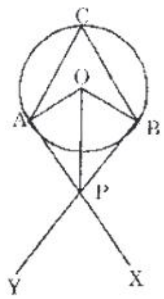
$$\angle BAC = 180^\circ - 140^\circ = 40^\circ$$

7.



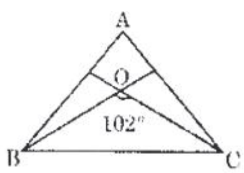
$$\angle A = 180^\circ - 60^\circ - 40^\circ = 80^\circ$$
$$\angle BAD = \frac{80}{2} = 40^\circ$$
$$BAE = 180^\circ - 60^\circ - 90^\circ = 30^\circ$$
$$\angle DAE = 40^\circ - 30^\circ = 10^\circ$$

8.



$$\angle ACB = 65^\circ$$
$$\angle AOB = 2 \times 65^\circ = 130^\circ$$
$$\angle OAP = 90^\circ; \angle AOP = 65^\circ$$
$$\therefore \quad \angle APO = 180^\circ - 90^\circ - 65^\circ$$
$$= 25^\circ$$

9.

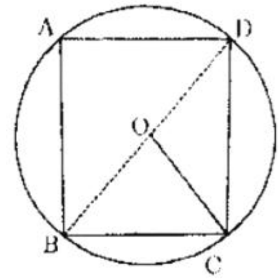


$$\angle A + \angle B + \angle C = 180^\circ$$
$$\frac{\angle B}{2} + \frac{\angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

In ΔBOC,

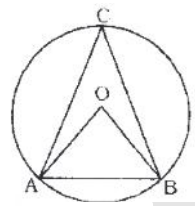
$$\angle BOC + \frac{\angle B}{2} + \frac{\angle C}{2} = 180^\circ$$
$$\Rightarrow \quad 102^\circ + 90^\circ - \frac{\angle A}{2} = 180^\circ$$
$$\Rightarrow \quad \frac{\angle A}{2} = 102^\circ + 90^\circ - 180^\circ = 12^\circ$$
$$\Rightarrow \quad \angle A = 24^\circ$$

10. **the angle subtended at the centre by an arc is twice to that of angle subtended at the circumference.**



$$\therefore \quad \angle CAD = \frac{1}{2}\angle COD = 70^\circ$$
$$\therefore \quad \angle BAD = 70^\circ + 40^\circ = 110^\circ$$
$$\therefore \quad \angle BCD = 180^\circ - 110^\circ = 70^\circ$$

11.



$\therefore \quad OA = OB = AB$

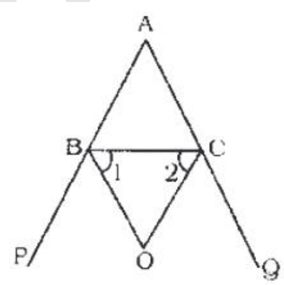
$\therefore \quad \Delta OAB$ is an equilateral triangle

$\therefore \quad \angle AOB = 60$

The angle subtended by an arc at the circumference is half of that at the centre

$$\therefore \quad \angle ACB = \frac{1}{2} \times 60^\circ = 30^\circ$$

12.



$$\angle ABC + \angle CBP = 180^\circ$$
$$\Rightarrow \quad \angle B + 2\angle = 180^\circ$$
$$\Rightarrow \quad 2\angle 1 = 180^\circ - \angle B$$
$$\Rightarrow \quad \angle 1 = 90^\circ - \frac{1}{2}\angle B$$

Again, $\quad \angle ACB + \angle QCB = 180^\circ$

$$\Rightarrow \quad \angle 2 = 90^\circ - \frac{1}{2}\angle C$$

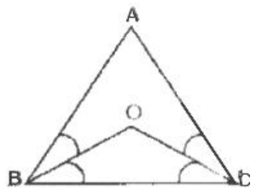
In ΔBOC,

$$\angle 1 + \angle 2 + \angle BOC = 180^\circ$$
$$\Rightarrow \quad 90^\circ - \frac{1}{2}\angle B + \angle 90^\circ = \frac{1}{2}$$
$$\angle C + \angle BOC = 180^\circ$$
$$\Rightarrow \quad \angle BOC = \frac{1}{2}(\angle B + \angle C)$$
$$= \frac{1}{2}(180^\circ - \angle A)$$
$$\Rightarrow \quad \angle BOC = 90^\circ - \frac{1}{2}\angle A$$

$\Rightarrow \quad 60^\circ = 90^\circ - \frac{1}{2}\angle A$

$\Rightarrow \quad \angle A = 60^\circ$

13.



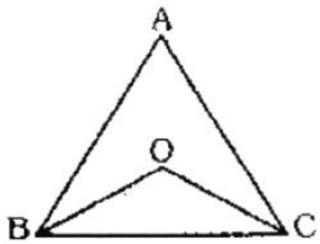
$\angle BAC = 80^\circ$

$\angle ABC + \angle ACB = 100^\circ$

$\angle OBC + \angle OCB = 50^\circ$

$\angle BOC = 180^\circ - 50^\circ = 130^\circ$

14.



$\angle BAC = 80^\circ$

$\therefore \quad \angle B + \angle C = 180^\circ - 80^\circ = 100^\circ$

$\frac{\angle B}{2} + \frac{\angle C}{2} = 50^\circ$

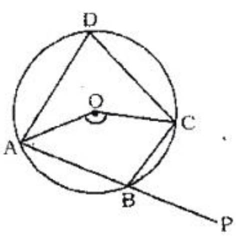
$\therefore \quad \angle OBC + \angle OCB = 50^\circ$

$\therefore \quad \angle BOC = 180^\circ - 50^\circ$

$= 130^\circ$

15. **It will be a right angle**

16.



$\angle AOC = 130^\circ$

$\angle ADC = \frac{1}{2} \times 130^\circ = 65^\circ$

$\angle PBC = \angle ADC = 65^\circ$

17. $\boldsymbol{3x + 3x + 4x = 180^\circ}$

$\Rightarrow \quad 9x = 180^\circ$

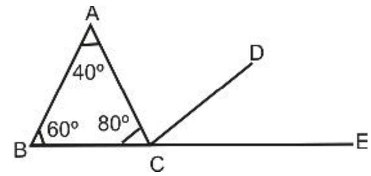
$\Rightarrow \quad x = 20^\circ$

angle of the triangle $40^\circ, 60^\circ$ and 80° $AB || CD$

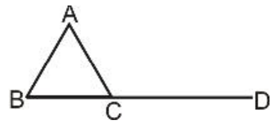
$\angle DCE = \angle ABC = 60^\circ$

$\angle ACB + \angle ACD + \angle DCE = 180^\circ$

$\angle ACD = 180^\circ - 120^\circ = 60^\circ$



18.



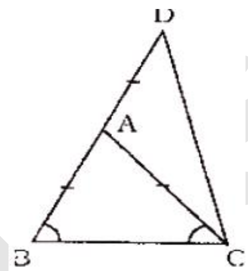
$\angle ACB = 180^\circ - 75^\circ - 45^\circ = 60^\circ$

$\angle ACD = 180^\circ - 60^\circ = 120^\circ = x$

$\frac{6}{3}$% of 60°

$= 60 \times \frac{120}{300} = 24^\circ$

19.



$\angle BC = \angle ACB = x$

$\angle BAC = 180^\circ - 2x$

$\angle AD = 180^\circ - 2x$

$\angle BAD = 180^\circ$

$\angle 50^\circ = (180^\circ - 2x) \times 2$

$\angle 50^\circ - 2x = 90^\circ$

$\angle x = 90^\circ$

$\angle$BCD

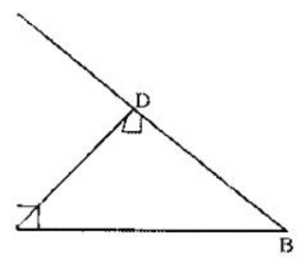
20. $\boldsymbol{\angle A + \angle B = 65^\circ}$

$\angle A = 180^\circ - 65^\circ = 115^\circ$

$\angle C = 140^\circ$

$\angle B = 140^\circ - 115^\circ - 25^\circ$

21.



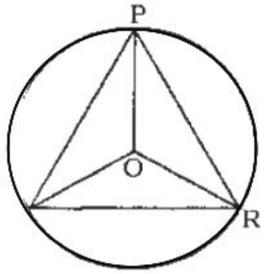
$\angle = 90^\circ, \angle C = 55^\circ$

$\angle B = 90^\circ - 55^\circ = 35^\circ$

$\angle CB = 90^\circ$

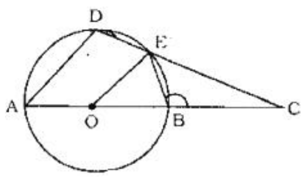
$\angle BAD = 90^\circ - 35^\circ = 55^\circ$

22.



$$PQR = 110^\circ$$
$$\angle OPR = 25^\circ$$
$$\angle QPR = 110^\circ \div 2 = 55^\circ$$
$$OR = OP$$
$$\angle OPR = \angle PRO = 25^\circ$$
$$\angle OQR = \angle ORQ = 70 / 2 = 35^\circ$$
$$\angle PRQ = 25^\circ + 35^\circ = 60^\circ$$

23.



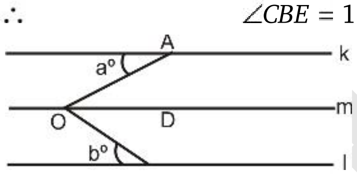
$$\angle AOE = 150^\circ$$
$$\angle DAO = 51^\circ$$
$$\angle EOB = 180^\circ - 150^\circ = 30^\circ$$

$OE = OB$

$\therefore \quad \angle OEB = \angle OBE = \dfrac{150}{2} = 75^\circ$

$\therefore \quad \angle CBE = 170^\circ - 75^\circ = 105^\circ$

24.



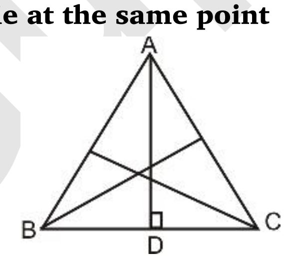
$$\angle BO / A = 45^\circ$$

$\Rightarrow \quad \angle AOD = a^\circ$

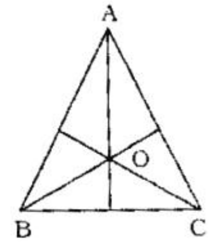
and $\quad \angle DOB = b^\circ$

$$a^\circ + b^\circ = \angle AOB = 45^\circ$$

25. **In equilateral triangle orthocentre and centroid lie at the same point**



26.



In equilateral triangle centre incentre, orthocentre coins the same point

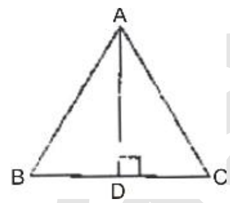
$\therefore \quad \dfrac{\text{Height}}{3} = \text{in radius}$

$\therefore \text{Height} = \text{Median} \quad = 3$

$= 9\,\text{cm}$

27. **Triangle will be equilateral**

28.



$$AD = b$$
$$BD = DC = x$$
$$Tan60^\circ = AD / BD$$
$$\sqrt{3} = b / x$$
$$x = \frac{b}{\sqrt{3}}$$

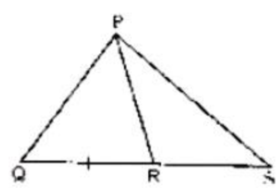
$\Rightarrow \quad BC = 2x = \dfrac{2b}{\sqrt{3}}$

area of the triangle

$$a = \frac{1}{2} \times \frac{2b}{\sqrt{3}} \times b$$

$\Rightarrow \quad \dfrac{b^2}{a} = \sqrt{3}$

29.

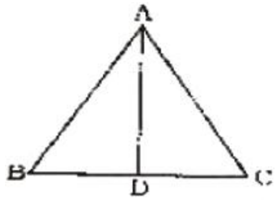


$$\angle PRQ = 60^\circ$$
$$\angle PRS = 180^\circ - 60^\circ = 120^\circ$$
$$\angle PSR = \angle RPS = 60^\circ$$
$$RS = PR$$

$\therefore \quad \angle PSR = \angle RPS$

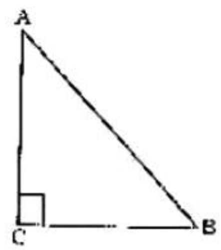
$\therefore \quad \angle PSR = \dfrac{60}{2} = 30^\circ$

30.



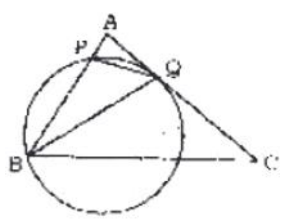
$$AB = AC = 2a \text{ units}$$
$$BC = a \text{ units}$$
$$BD = DC = \frac{a}{2} \text{ units}$$
$$AD = \sqrt{AB^2 - BD^2}$$
$$= \sqrt{4a^2 - \frac{a^2}{4}} = \sqrt{\frac{15a^2}{4}}$$
$$= \frac{\sqrt{15}}{2} . a \text{ units}$$

31.



$$AC = BC = 5 \text{ cm}$$
$$\therefore \quad AB = \sqrt{AC^2 + BC^2}$$
$$= \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

32.

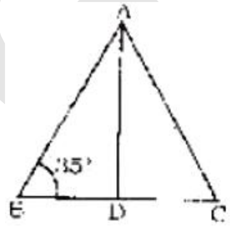


$$AB = AC = 3x$$
$$AQ = QC = x$$

AB is a sccant

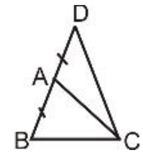
$$\Rightarrow \quad AP \times AB = AQ^3$$
$$\Rightarrow \quad AP \times 2x = x^2$$
$$\Rightarrow \quad AP = \frac{x}{9}$$
$$\therefore \quad \frac{AP}{AB} = \frac{x}{2 \times 2x} = \frac{1}{4}$$

33.



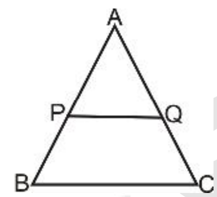
$$AB = AC$$
$$\therefore \quad \angle ABC = \angle ACB = 35^\circ$$
$$\therefore \quad \angle ADB = 90^\circ$$
$$\therefore \quad \angle BAD = 55^\circ$$

34.



$$AB = AC = AD$$
$$\angle ABC = \angle ACB = 30^\circ$$
$$\angle BAC = 180^\circ - 60^\circ = 120^\circ$$
$$\angle DAC = 180^\circ - 120^\circ = 60^\circ$$
$$\angle ADC + \angle ACD = 120^\circ$$
$$\angle ACD = 120^\circ / 2 = 60^\circ$$
$$\angle BCD = \angle ACB + \angle ACD$$
$$= 30^\circ + 60^\circ = 90^\circ$$

35.



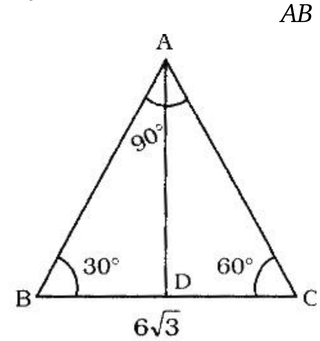
$\Delta APQ \sim \Delta ABC$

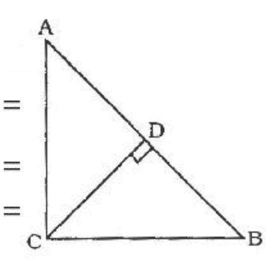
$$\therefore \quad \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

Now, $\quad \frac{AB}{PB} = \frac{3}{1}$

$$\Rightarrow \quad \frac{AB}{AB - AP} = \frac{3}{1}$$
$$\Rightarrow \quad \frac{AB - AP}{AB} = \frac{1}{3}$$
$$\Rightarrow \quad 1 - \frac{AP}{AB} = \frac{1}{3}$$
$$\Rightarrow \quad \frac{AP}{AB} = 1 - \frac{1}{3} = \frac{2}{3} = \frac{PQ}{BC}$$

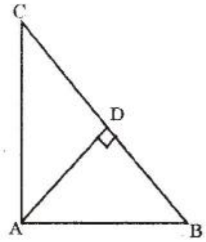
36.





$$30^\circ = \frac{AC}{BC}$$
$$= \quad \frac{1}{2} = \frac{AC}{6\sqrt{3}}$$
$$= \quad AC = 3\sqrt{3}$$
$$= \quad 60^\circ = \frac{AD}{AC}$$
$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AD}{3\sqrt{3}}$$
$$\Rightarrow \quad AD = \frac{3\sqrt{3} \times \sqrt{3}}{2} = 4.5 \text{ cm}$$

37.



$$\angle BAC = 90^\circ$$
$$AB = \sqrt{AD^2 + BD^2}$$
$$= \sqrt{36+16} = \sqrt{52}\text{ cm}$$

ΔABD and ΔABC are similar

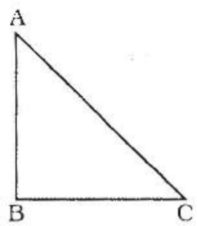
$\therefore$ $$\frac{AB}{BC} = \frac{BD}{AB}$$

$\Rightarrow$ $$AB^2 = BC \times BD$$

$\Rightarrow$ $$52 = BC \times 4$$

$\Rightarrow$ $$BC = \frac{52}{4} = 13\text{ cm}$$

38.



$$= AC^2 = 2AB \times BC$$
$$= AB^2 + BC^2 = 2AB \times BC$$
$$= (AB - BC)^2 = 0$$
$$= AB = BC$$
$$\angle BAC = \angle ACB = 45^\circ$$

39.

$$\angle BAC = 90^\circ$$
$$\angle ADC = 90^\circ$$
$$BC = 8\text{ cm},\ AC = 6\text{ cm}$$
$$AB = \sqrt{8^2 - 6^2}$$
$$= \sqrt{14 \times 2} = 2\sqrt{7}\text{ cm}$$

Area ΔABC

$$= \frac{1}{2} \times BC \times AD, \frac{1}{2} \times AB \times AC$$

$\Rightarrow$ $$8 \times 2D = 2\sqrt{7} \times 6$$

$\Rightarrow$ $$AD = \frac{3\sqrt{7}}{2}\text{ cm}$$

$$CD = \sqrt{6^2 - \left(\frac{3\sqrt{7}}{2}\right)^2}$$
$$= \sqrt{36 - \frac{63}{4}}$$
$$= \sqrt{\frac{144-63}{4}}$$
$$\sqrt{\frac{81}{4}} = \frac{9}{2}$$
$$\frac{\Delta ABC}{\Delta ACD} = \frac{AB \times AC}{CD \times AD} = \frac{2\sqrt{7} \times 6}{\frac{9}{3} \times \frac{3\sqrt{7}}{2}}$$
$$= \frac{2\sqrt{7} \times 6 \times 4}{9 \times 3 \times \sqrt{7}} = 16{:}9$$

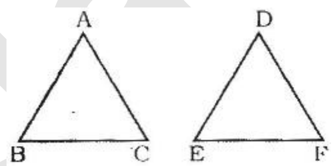
40. $$\frac{\Delta \boldsymbol{ABC}}{\Delta \boldsymbol{DEF}} = \frac{\boldsymbol{AB^2}}{\boldsymbol{DE^2}}$$

$\Rightarrow$ $$\frac{20}{45} = \frac{AB^2}{DE^2}$$

$\Rightarrow$ $$DE^2 = \frac{45 \times 25}{20} = \frac{225}{4}$$

$\therefore$ $$DE = \frac{15}{2} = 7.5\text{ cm}$$

41.



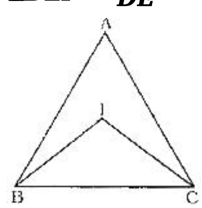
$\Delta ABC \sim \Delta DEF$

$\therefore$ $$\frac{\Delta ABC}{\Delta DEF} = \frac{3^2}{4^2} \Rightarrow \frac{54}{\Delta DEF} = \frac{9}{16}$$

$\Rightarrow$ $$\Delta DEF = \frac{16 \times 54}{9}$$
$$= 96\text{ sq. cm}$$

42. $$\frac{\Delta \boldsymbol{ABC}}{\Delta \boldsymbol{DEF}} = \frac{\boldsymbol{AB^2}}{\boldsymbol{DE^2}} = \frac{\mathbf{100}}{\mathbf{64}} = \frac{\mathbf{25}}{\mathbf{16}}$$

43.



$$\angle B + \angle C = 180 - 50 = 130^\circ$$

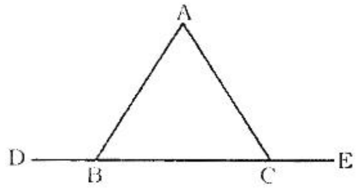
In ΔBIC

$$\angle IBC + \angle ICN + \angle BIC = 180^\circ$$

$\Rightarrow$ $$\frac{\angle B}{2} + \frac{\angle C}{2} + \angle BIC = 180^\circ$$

$\Rightarrow$ $$\angle BIC = 180^\circ - \frac{1}{2}(\angle B + \angle C)$$
$$= 180^\circ - \frac{130}{2}$$
$$= 180^\circ - 65^\circ = 115^\circ$$

44.



$$\angle ABD = 120^\circ$$

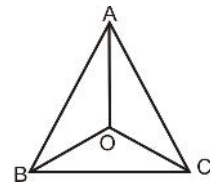
$$\therefore \quad \angle ABC = 180^\circ - 120^\circ = 60^\circ$$

$$\angle ACE = 105^\circ$$

$$\therefore \quad \angle ACB = 180^\circ - 105^\circ = 75^\circ$$

$$\therefore \quad \angle BAC = 180^\circ - 60^\circ - 60^\circ = 45^\circ$$

45.



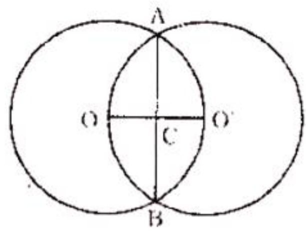
$$\angle ABD = \pi - B \angle ACE = \pi - C$$

$$\angle ABD + \angle ACE = 2\pi - (B + C)$$

$$= 2\pi - (\pi - A) = \pi + A$$

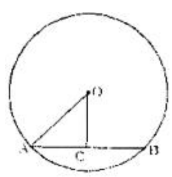
SIMPLE CIRCLE AND TANGENT SOLUTION

1.



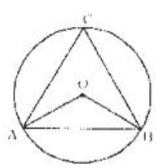
$$OC = 2\text{ cm}$$
$$OA = 4\text{ cm}$$
$$AC = \sqrt{4^2 - 2^2} = \sqrt{16-4}$$
$$= \sqrt{12} = 2\sqrt{3}$$
$$\therefore \quad AB = 4\sqrt{3}\text{ cm}$$

2.



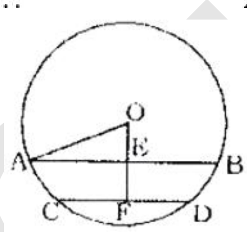
$$AB = CB = 4\text{ cm}$$
$$OC = 3\text{ cm}$$
$$\therefore \quad OA = \sqrt{OC^2 + CA^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5\text{ cm}$$

3.



$$AO = OB = AB$$
$$\therefore \quad \angle AOB = 60^\circ$$
$$\therefore \quad \angle ACB = 30^\circ$$

4.



Let

$$OE = x\text{ cm}$$
$$\therefore \quad OF = (x+1)\text{ cm}$$
$$OA = OC = r\text{ cm}$$
$$AE = 4\text{ cm}$$
$$CF = 3$$

From ΔOAK

$$OA^2 = AE^2 + OE^2$$
$$\Rightarrow \quad r^2 = 16 + x^2$$
$$\Rightarrow \quad x^2 = r^2 - 16$$

From ΔOCF,

$$(x+1)^2 = r^2 - 9$$

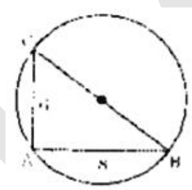
By equation (ii) (i),

$$(x+1)^2 - x^2 = x^2 - 9$$
$$\Rightarrow \quad 2x + 1 = 7$$
$$\Rightarrow \quad x = 3$$

$\therefore$ From equation (i)

$$9 = r - 16$$
$$\Rightarrow \quad r^2 = 12$$

5.



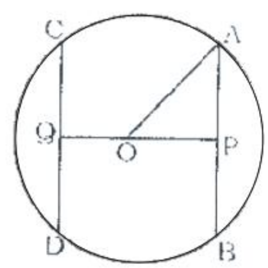
$$\angle BAC = 90^\circ$$

$\therefore BC$ is the diameter

$$\therefore \quad BC = \sqrt{AB^2 + AC^2}$$
$$= \sqrt{8^2 + 6^2} = \sqrt{64-3}$$
$$= \sqrt{100} = 10\text{ cm}$$

$\therefore$ Radius of the circle

6.



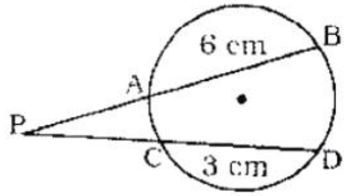
$$AB = CD$$
$$OP = OQ$$

From ΔOAP,

$$OP = \sqrt{OA^2 - AP^2} - \sqrt{5^2 - 4}$$
$$= \sqrt{25-16}$$
$$= \sqrt{9} - 3\text{ cm}$$
$$\therefore \quad QP = 2 \times QP = 6\text{ cm}$$

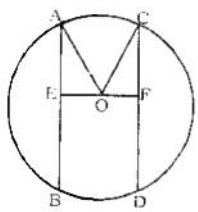
7. **The largest chord of a circle is its diameter.**

8.



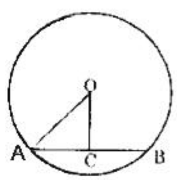
$$
\begin{aligned}
AB &= 6\text{ cm}\\
CD &= 3\text{ cm}\\
PD &= 5\text{ cm}\\
PB &= ?\\
PA \times PB &= PC \times PD\\
\Rightarrow \quad (PB-6)PB &= 2\times 5\\
\Rightarrow \quad PB^2 - 6PB - 10 &= 0\\
\Rightarrow \quad PB &= \frac{6 = \sqrt{36+40}}{2}\\
&= \frac{6 \pm \sqrt{76}}{2}\\
&= \frac{6+87}{2} = 7.35
\end{aligned}
$$

9.



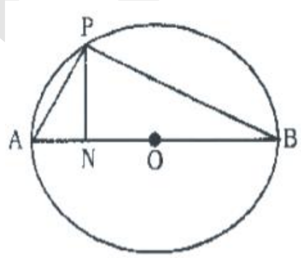
$$
\begin{aligned}
AB &= 24\\
AR = EB &= 12\text{ cm}\\
OE &= \sqrt{OA^2 - AE^2}\\
&= \sqrt{15^2 - 12^2}\\
&= \sqrt{225-144} = \sqrt{81}\\
&= 9\text{ cm}\\
\therefore \quad OF &= 21 - 9 = 12\text{ cm}\\
\therefore \quad CD &= 2\times 9 = 18\text{ cm}
\end{aligned}
$$

11.



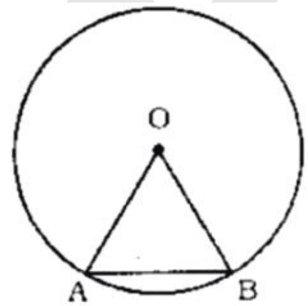
$$
\begin{aligned}
5C &= 4\text{ cm}\\
&= 3\text{ cm}\\
&= \sqrt{AC^2 + OC^2}\\
&= \sqrt{16+9} = \sqrt{25} = 5\text{ cm}
\end{aligned}
$$

12.



$$
\begin{aligned}
AB &= 14\text{ cm}\\
PB &= 12\text{ cm}\\
\angle APB &= 90^\circ\\
\therefore \quad AP &= \sqrt{14^2 - 12^2}\\
&= \sqrt{(14+12)(14-12)}\\
&= \sqrt{26\times 2} = \sqrt{52}\\
ON &= x\\
\therefore \quad AN &= 7 - x : BN = 7 + x\\
\therefore \text{ From } \quad \Delta PAN.\, PN^2 &= AP^2 - AN^2\\
&= 52 - (7-x)^2\\
\Rightarrow \quad 52 - (59 - 14x + x^2) &= 144 - (49 + 14x + x^2)\\
&= 14x - x^2\\
\Rightarrow \quad 28x &= 144 - 52 - 92\\
\Rightarrow \quad x &= \frac{92}{28} = \frac{23}{7}\\
\therefore \quad BN &= 7 + x\\
&= 7 + \frac{23}{7} = \frac{49+23}{7} = \frac{72}{7}\\
&= 10\frac{2}{7}\text{ cm}
\end{aligned}
$$

13.

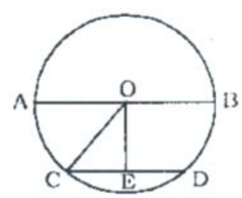


$$OA = OB = AB$$

$\therefore \Delta OAB$ is an equilateral triangle

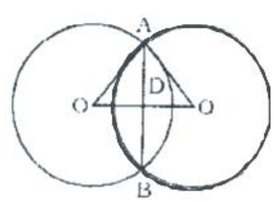
$$\therefore \quad \angle AOB = 60^\circ$$

14.



$$
\begin{aligned}
OC = \text{radius} &= 10\text{cm}\\
CE = ED &= 6\text{cm}\\
\therefore \quad OE &= \sqrt{OC^2 - CE^2}\\
&= \sqrt{10^2 - 6^2} = \sqrt{100-36}\\
&= \sqrt{64} = 8\text{ cm}
\end{aligned}
$$

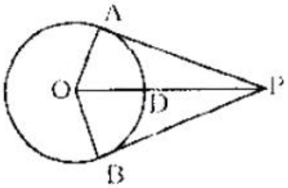
15.



$$
\begin{aligned}
OC &= \sqrt{15^2 - 12^2}\\
&= \sqrt{225-144}\\
&= \sqrt{51} = 9
\end{aligned}
$$

$$= \sqrt{13^2 - 12^2}$$
$$= \sqrt{69 - 144} = \sqrt{25} \times 5$$
$$= 72^\circ$$
$$= 72 \times \frac{r}{180} \text{ radians}$$

16.



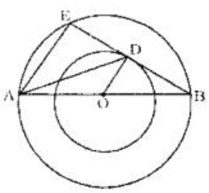
$$OA = OE = r$$
$$OP = 2r$$
$$AP = PB$$
$$= \sqrt{4r^2 - r^2} = \sqrt{3}r$$

sin APO $$= \frac{OA}{OP} = \frac{r}{2r} = \frac{1}{2}$$

$$\angle APO = 30^\circ$$

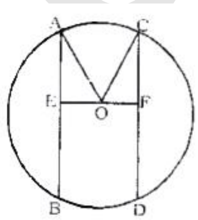
$\therefore$ $$\angle APB = 60^\circ$$

17.



$$\angle ODB = 90^\circ$$
$$OD = 8 \text{ cm}$$
$$OB = 13 \text{ cm}$$

$\therefore$ $$BD = \sqrt{13^1 - 8^2}$$
$$= \sqrt{169 - 64}$$
$$= \sqrt{105} \text{ cm}$$
$$AE = 16 \text{ cm}$$
$$\angle AED = 90^\circ$$
$$AD = \sqrt{AE^2 + DE^2}$$
$$AD = \sqrt{256 + 105} = \sqrt{361}$$
$$= 19 \text{ cm}$$



$OE \perp AB$and $OF \perp CD$

$$AE = EB = 5 \text{ cm}$$
$$CF = FD = 12 \text{ cm}$$
$$AO = OC = 13 \text{ cm}$$

From ΔAOE,

$$OE = \sqrt{13^2 - 5^2} = \sqrt{169 - 25}$$
$$= \sqrt{144} = 12 \text{ cm}$$

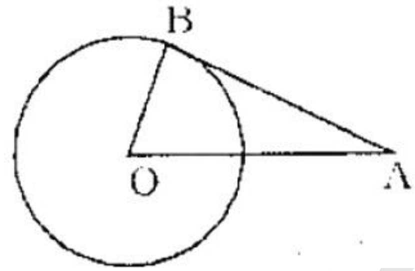
From ΔCOF

$$OF = \sqrt{13^2 - 12^2} = \sqrt{25} = 5$$

cm

$\therefore$ $$EF = OE + OF = 17 \text{ cm}$$

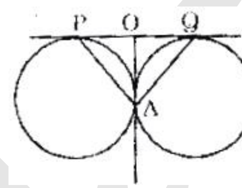
18.



$$\angle OBA = 90^\circ$$
$$OA = 5$$
$$OB = 4$$

$\therefore$ $$AB = \sqrt{OA^2 - OB^2}$$
$$= \sqrt{25 - 16} = \sqrt{9} = 3 \text{ cm}$$

19.

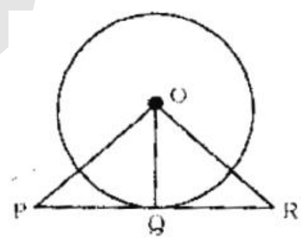


AO is perpendicular to PQ

$$OA = OP - OQ$$
$$\angle OPA = \angle OAP = \angle OQA = 45^\circ$$

$\therefore$ $$\angle PAQ = 90^\circ$$

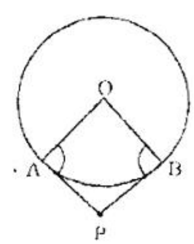
20.



$$QR = \sqrt{OR^2 - OB^2}$$
$$= \sqrt{5^2 - 4^2}$$
$$= \sqrt{9} = 3 \text{ cm}$$

$\therefore$ $$PR = PQ + QR$$
$$= \frac{16}{3} + 3 = \frac{23}{4}$$

21.



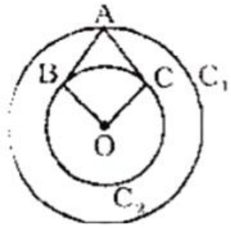
$$\angle OAP = \angle OBP = 90^\circ$$
$$\angle AOB + \angle APB = 180^\circ$$

$\Rightarrow$ $$5\angle APB - \angle APB = 180^\circ$$

$\Rightarrow$ $$6\angle APB = 180^\circ$$

$\Rightarrow$ $$\angle APB = 30^\circ$$

23.

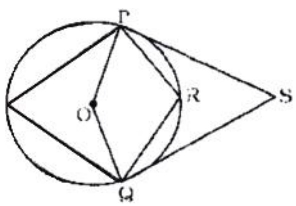


AC tangents from the

$$OC = 3\,\text{cm}$$
$$= 12\,\text{cm}$$
$$ABO = 90^\circ$$
$$AB = \sqrt{12^2 - 3^2}$$
$$\frac{15}{2} \times 3 \times 3\sqrt{15} = \frac{9\sqrt{15}}{2}$$

Area of $OABC$ $= 9\sqrt{15}$ sq.cm

24.



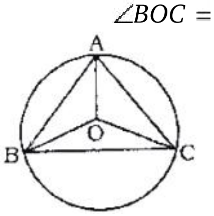
$$\angle QPS = \angle OQS = 90^\circ$$
$$\angle PSQ = 20^\circ$$
$$\angle POQ = 160^\circ$$
$$\angle PTQ = 80^\circ$$

$\angle QT$ is a concyclic quadrilateral

$$\angle PRQ = 180^\circ - 80^\circ$$
$$= 100^\circ$$

26. $\because$ $\boldsymbol{\angle BAC = 85^\circ}$

$$\angle BOC = 2 \times 85^\circ = 170^\circ$$

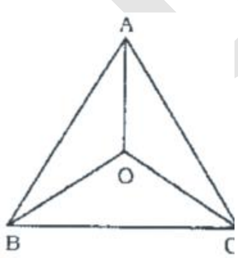


$\therefore$ $\angle OBC = \angle OCB = 5^\circ$

$\therefore$ $\angle OCA = \angle OAC = 75^\circ - 5^\circ$

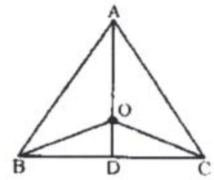
$$= 70^\circ$$

27.



$$\angle BOC = 90^\circ + \frac{1}{2}\angle BAC$$
$$= 90^\circ + 15^\circ = 105^\circ$$

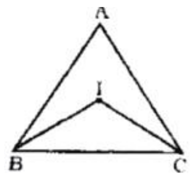
28.



BO is the internal bisector of $\angle B$

$$\angle ODB = 90^\circ; \angle BOD = 15^\circ$$
$$\angle OBD = 180^\circ - 90^\circ - 15^\circ = 75^\circ$$
$$\angle ABC = 2 \times 75^\circ = 150^\circ$$

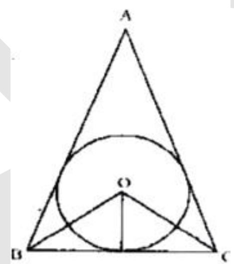
29.



$$\angle IBC = \frac{1}{2}\angle ABC = 30^\circ$$
$$\angle ICB = \frac{1}{2}\angle ACB = 25^\circ$$

$\therefore$ $\angle BIC = 180^\circ - 30^\circ - 25^\circ$

32.



$$\angle BOC = 90^\circ + \frac{A}{2}$$

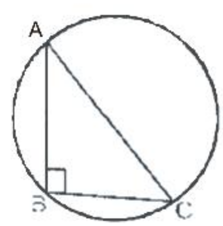
$\Rightarrow$ $110 = 90^\circ = \frac{A}{2}$

$\Rightarrow$ $A = 2 \times 20 = 40^\circ$

33. The right bisectors of sides meet at a point called circumcentre.

34. In an equilateral triangle centriod, incentre etc. ie at the same point

35.



$$3^2 + 4^2 = 5^2$$

triangle ABC is a right angled triangle

$\angle ABC = 90^\circ =$ angle at the circumference diameter of circle $= 5\,\text{cm}$

circum radius $= 2.5$

36. $\boldsymbol{OA = OB = OC}$

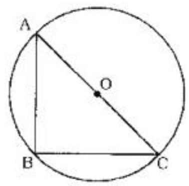
$$\angle BID = \angle ABC$$

$\Rightarrow$ $x = y$

$$\angle BOD = 2\angle BAD$$

$\therefore$ $$\frac{z+x}{y} = \frac{3y}{y} = 3$$

37.



$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{15^2 + 20^2} = \sqrt{225 + 400}$$

$$= \sqrt{625} = 25 \text{ cm}$$

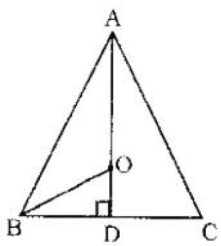
$$\angle ABC = 90^\circ$$

$\therefore AC =$ diameter

$\therefore$ Circum radius(OA) $= \dfrac{25}{2}$

$$= 12.5 \text{ cm}$$

38.



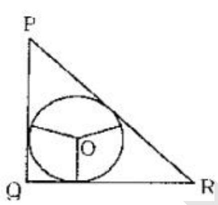
Circum-radius of equilateral triangle

$$= \frac{2}{3} \times \text{height}$$

$\therefore$ $$8 = \frac{2}{3} \times \text{height}$$

$\therefore$ $$\text{Height} = \frac{8 \times 3}{2} = 12 \text{ cm}$$

39.



$$PR^2 = PQ^2 + PR^2$$

$$= 3^2 + 4^2 = 25$$

$\therefore$ $$PR = \sqrt{25}$$

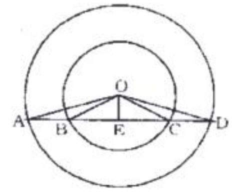
$$= \sqrt{5} \text{ cm}$$

r

$$= \frac{\text{Area of triangle}}{\text{Semi - perimeter of triangle}}$$

$$= \frac{\frac{1}{2} \times 3 \times 4}{\frac{3+4+5}{2}} = \frac{6}{6} = 1 \text{ cm}$$

40.



$$BE = EC = 6 \text{ cm}$$

$$OB = 10 \text{ cm}$$

$$OA = 17 \text{ cm}$$

From ΔOBE

$$OE = \sqrt{OB^2 - BE^2}$$

$$= \sqrt{10^2 - 6^2}$$

$$= \sqrt{16 \times 4} = 8 \text{cm}$$

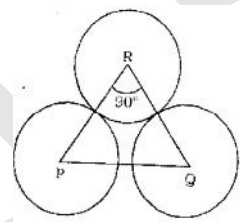
From ΔOAE

$$AE = \sqrt{OA^2 - OE^2}$$

$$= \sqrt{17^2 - 8^2} = \sqrt{25 \times 9} = 15 \text{ cm}$$

$\therefore$ $$AD = 2AE = 2 \times 15 = 30 \text{ cm}$$

41.



$$\angle PRQ = 90^\circ$$

$$PR = 2 + x$$

$$PQ = 17$$

$$RQ = 9 + x$$

$\therefore$ $$PQ^2 = PR^2 + RQ^2$$

$\Rightarrow$ $$17^2 = (2+x)^2 + (9+x)^2$$

$\Rightarrow$ $$280 = 4 + 4x + x^2 + 81 + 18x$$

$\Rightarrow$ $$289 = 2x^2 + 22x + 85$$

$\Rightarrow$ $$2x^2 + 22x + 85 - 289 = 0$$

$\Rightarrow$ $$2x^2 + 22x - 204 = 0$$

$\Rightarrow$ $$x^2 + 11x - 102 = 0$$

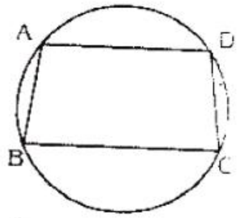
$\Rightarrow$ $$x^2 + 11x - 6x - 102 = 0$$

$\Rightarrow$ $$x(x+17) - 6(x+18) = 0$$

$\Rightarrow$ $$(x-6)(x+17) = 0$$

$\Rightarrow$ $$x = 6 \text{as} \quad x \neq -17$$

42.

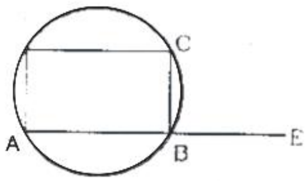


$$\angle ABC + \angle CDA = 180^\circ$$

$\therefore$ $$\angle CDA = 180^\circ - 70^\circ$$

$\therefore$ $$\angle BCD = 180^\circ - 110^\circ$$

43.



$\angle ABC + \angle ADC = 180^\circ$

$\angle CBE = 50^\circ$

$\angle ABC = 180^\circ - 50^\circ = 130^\circ$

$\angle ADC = 180^\circ - 130^\circ = 50^\circ$

POLYGON ANSWER KEY

1. **Step 1 : Write down the formula $(n-2)\times 180^\circ$**

Step 2 : plug in the values

$$(7-2)\times 180^\circ = 5\times 180^\circ = 900^\circ$$

The sum of the interior angles of a heptagon (7 –sides) is 900°

2. **Step 1 : Write down the formula $\dfrac{(n-2)\times 180^\circ}{n}$**

Step 2 : Plug in the values

$$\frac{(8-2)\times 180^\circ}{8} = 135^\circ$$

Each interior angle of an otagon (8-sided) is 135°

The answer is $180^\circ - 45^\circ = 135^\circ$

A regular polygon has equal exterior angles of 72°

3. **The sum of interior angles of a polygon of n sides is given by**

$$(2n-4)\times\frac{\pi}{4}$$

$$(2n-4)\times\frac{\pi}{2} = 1620\times\frac{\pi}{180}$$

$$(2n-4) = \frac{1620\times 2}{180} = (2n-4) = \frac{3240}{180}$$

$$2n-4 = 18$$

$$2n = 22, \Rightarrow n = 11$$

4. **Let n be the number of sides of the polygon. Interior angle = $8\times$ Exterior angles**

$$\frac{(2n-4)\times\frac{\pi}{2}}{n} = 8\times\frac{2\pi}{n}$$

$$n-2 = 16 \Rightarrow n = 18$$

5. **Each interior angle of regular polygon $= 2n - 4.90/n$**

Each external angle of regular polygon $= 360/n$

By question, $= 2n - 4.90/n = 2.360/n$

After solving above question we get $n = 6$

6. **Sum of interior Angles in a Polygon= $180(n-2)$**

Sum of interior Angles in a Polygon

$$= 180(5-2)$$

Sum of interior Angles in a Polygon

$$= 180(3)$$

Sum of interior Angles in a Polygon

$$= 540$$

Now you have $\times$ parts that make up his degree.

$$x = 2+3+3+5+5$$

$$x = 18$$

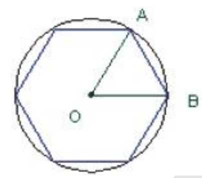
$540/18 =$ The Degree of one part

$30 =$ The Degree of One part

And the smallest angle has $2x$ so 2 time 30 equals to 60 degree

Therefore the smallest angle is 60 degree.

7.



Angle AOB is given by

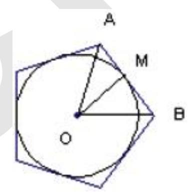
Angle (AOB) $= 360^\circ / 6 = 60^\circ$

Since $OA = OB = 10$ cm triangle OAB is isosceles which given

So all three angles of the triangle are equal and therefore it is an equailateral triangle. Hence

$$AB = OA = OB = 10 \text{ cm}$$

8.



Let t be the size of angle AOB, hence

$t = 360^\circ / 5 = 72^\circ$

The polygon is regular and $OA = OB$ Let M be the midpoint of AB so that OM is perpendicular to AB, OM is the radius of the inscribedcircle and is equal to 6 cm.

Right angle trigonometry gives

$$\tan(1/2) = MB/OM$$

The side of the pentagon is twice MB, hence

side of pentagon $= 2OM\tan(t/2) = 8.7$ cm (answer rounded to two decimal places)

9. **A dodecagon is a regular polygon with 12 sides and the central angle t opposite one side of the polygon is given by.**

$t = 360^\circ / 12 = 30^\circ$

We now use the formula for the area when the side of the regular polygon is known.

$$\text{Area} = (1/4)nx^2\cot(180^\circ/n)$$

$$\text{Set } n = 12 \text{ and } x = 16 \text{ mm}$$

$$\text{area} = (1/4)(12)(6)^2$$

$$\cot(180^\circ/12)$$

$$= 403.1 \text{ mm}^2$$ (appoximated to 1 decimal place)

10. Let the smallest side of the plygon be a The largest side of the polygon =20 a

Since the polygon has 25 sides of the polygon are respectively $a, a+d, a+2d\ldots, a+23d, a+24d$; d being the common difference.

$a+24d = 20a \Rightarrow 19a = 24d$

Sum of the lengths of the sides = 2100

$$a+(a+d)+\ldots+(a+24d) = 2100$$
$$25a + d(1+2+\ldots24) = 2100$$
$$25a + d\left[\frac{(2424+1)}{2}\right] = 2100$$
$$25a + 300d = 2100$$
$$25 \times \frac{24d}{19} + 300d = 2100$$
$$\frac{600d}{19} + 300d = 2100$$
$$\frac{6d}{19} + 3d$$

$= 21 \Rightarrow 63d = 19 \times 21 \Rightarrow d = 19/3$

$$19a = \frac{24 \times 19}{3} = 8 \times 19, a = 8$$

Smallest side $= 8$ cm

And the common difference

$$= 19/3 = 6\frac{1}{3} \text{ cm}$$

11. Interior and exterior angle are always

Supplementery i.e., interior angle+ exterior angle

$= 180$

Their ratio is given 2 : 1

So that exterior angle of the polygon

$= 180 \times 1/3$

But sum of the exterior angle of a polygon is always

$= 360^\circ$

Therefore the no. of sides $= 360^\circ / 6 = 6$

12. Let there be n side polygon, each side $2p/n$

$A = n \times$ area of triangle whose side is $2P/n$ and altitude 'r'.

$$A = n \times \frac{1}{2} \times 2\frac{p}{n} \times r$$

$\therefore$ $r = A/P$

13. Let n be number of sides of polygon

Sum of the interior angles of a polygon of n sides

$$n \times \frac{5\pi}{6} = (n-2) \times \pi \Rightarrow n = 12$$

14. Sum of the interior angle = $(n-2)180^\circ$

So, sum of the interior angles of a six sides polygon

$= 6 - 2 \times 180^\circ = 720$

Sum of the interior angles of a eight sided polygon

$= (8-2) \times 180^\circ = 1080^\circ$

and sum of the interior angles of a ten sides polygon.

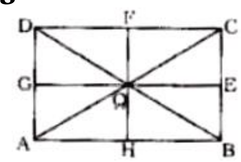
$= (10-2) \times 180 = 1440^\circ$

J ANSWER KEY QUADRILATERAL

1. (a)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (b)
11. (c)	**12.** (b)	**13.** (b)	**14.** (b)						

HINT & SOLUTIONS

1. Using Pythagoras theorem

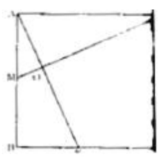


$$QD^2 + QB^2 = AQ^2 + QC^2$$

$\Rightarrow \quad QD^2 = 23 - 16 = 18$

$\Rightarrow \quad QD = \sqrt{18} = 3\sqrt{2}$ cm

2.



If $AB = 2x$, then $BN = x$

$$AN = \sqrt{4x^2 + x^2} = \sqrt{5}x$$

Similarly $\quad MD = \sqrt{4x^2 + x^2} = \sqrt{5}x$

3. sum of in interior angles of a regular polygon of n sides $= (2n - 4) \times 90^\circ$

So that $\quad (2n - 4) \times 90^\circ = 1080^\circ$

$$2n - 4 = 1080 \div 90 = 12$$

$$2n = 12 + 4 = 16$$

$$n = 8,$$

4. Let the number of sides be $5x$ and $4x$ respectively.

$\therefore \quad \dfrac{(2 \times 5x - 4)90}{5x} - \dfrac{(2 \times 4x - 4) \times 90^\circ}{4x} = 6^\circ$

$$\left[\begin{array}{l} \textit{Each interiro angle} \\ = \left(\dfrac{2n-4}{n}\right) \times 90^\circ \end{array}\right]$$

$\Rightarrow (10x - 4) \times 360^\circ - (8x - 4) \times 450^\circ = 20x \times 6^\circ$

$$450^\circ = 20x \times 6^\circ$$

$(10x - 4) \times 12 - (8x - 4)15 = 4x$

$\Rightarrow \quad 120x - 48 - 120x + 60 = 4x$

$\Rightarrow \quad 4x = 12 \quad \Rightarrow \quad x = 3$

$\therefore$ Number of sides

$$= 15 \text{ and } 12$$

5. Each interior angle of polygon of n sides.

$$= \left(\frac{2n-4}{n}\right) \times 90^\circ$$

$$\text{Each exteriror angle} = \frac{360^\circ}{n}$$

$\therefore \quad \dfrac{2n-4}{n} \times 90^\circ = \dfrac{2 \times 360}{n}$

$\Rightarrow \quad 2n - 4 = 8$

$\Rightarrow \quad 2n = 12$

$\Rightarrow \quad n = 6 =$ Number of sides

6. If the number of slides of regular polygon be n, then $(2n - 4) \times 90^\circ = 1440^\circ$

$\Rightarrow \quad 2n - 4 = \dfrac{1440}{90} = 16$

$\Rightarrow \quad 2n - 4 = 16$

$\Rightarrow \quad 2n = 20$

$\Rightarrow \quad n = 10$

7. If the number of sides of regular polygon be n, then $\dfrac{(2n-4) \times 90^\circ}{n} = 150^\circ$

$\Rightarrow \quad 3(2n - 4) = 5n$

$\Rightarrow \quad 6n - 12 = 5n \quad \Rightarrow \quad n = 12$

8. Each interior angle

$$= \frac{(2n-4) \times 90^\circ}{n}$$

$\therefore \quad \dfrac{\dfrac{(2n-4) \times 90^\circ}{n}}{\dfrac{(4n-4) \times 90^\circ}{2n}} = \dfrac{2}{3}$

$\Rightarrow \quad \dfrac{(2n-4) \times 2}{4n-4} = \dfrac{2}{3}$

$\Rightarrow \quad \dfrac{2n-4}{4n-4} = \dfrac{1}{3}$

$\Rightarrow \quad 6n - 12 = 4n - 4$

$\Rightarrow \quad 6n - 4n = 12 - 4 = 8$

$\Rightarrow \quad 2n = 8 \quad \Rightarrow \quad n = 4$

9. Sum of interior angles

$$= (2n - 4) \times 90^\circ$$

Sum of exterior angles $= 360^\circ$

$\therefore \quad (2n - 4) \times 90^\circ = 360^\circ \times 2$

$\Rightarrow \quad 2n - 4 = 2 \times 360^\circ \div 90 = 8$

$\Rightarrow \quad 2n - 4 = 8 \quad \Rightarrow \quad 2n = 12 \Rightarrow n = 6$

10. Each interior angle

$$= \left(\frac{2n-4}{n}\right) \times 90^\circ$$

$$\therefore \quad \frac{(2n-4)\times 90^\circ}{n} = 105^\circ$$

$$\Rightarrow \quad (12n-4)\times 6 = 7n$$

$$\Rightarrow \quad 12n - 24 = 7n$$

$$\Rightarrow \quad 5n = 24$$

$$\Rightarrow \quad n = \frac{24}{5} \text{ Which is impossible.}$$

11. Let the number of sides be $5x$ and $6x$ respectively.

$$\therefore \quad \frac{\frac{10x-4}{5x}}{\frac{12x-4}{6x}} = \frac{24}{25}$$

$$\left[\begin{array}{l} Each \ \text{int}\,eriro \ angle \\ = \dfrac{(2n-4)90^\circ}{n} \end{array}\right]$$

$$\Rightarrow \quad \frac{5x-2}{5}\times\frac{6}{6x-2} = \frac{24}{25}$$

$$\Rightarrow \quad \frac{5x-2}{6x-2} = \frac{4}{5}$$

$$\Rightarrow \quad 25x - 10 = 24x - 8$$

$$\Rightarrow \quad x = 10 - 8 = 2$$

$\therefore$ Number of sides = 10 and 12.

12. Let the number of sides of regular polygon be n

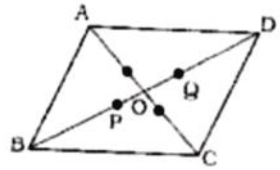
$$\therefore \quad \left(\frac{2n-4}{n}\right)\times 90^\circ = 2\times\frac{360}{n}$$

$$\Rightarrow \quad (2n-4) = 8$$

$$\Rightarrow \quad 2n = 12$$

$$\Rightarrow \quad n = 6$$

13.



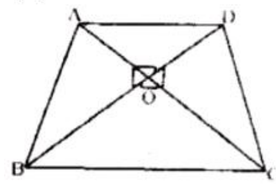
Centroid is the point where medians intersect. Diagnals of parallelogram bisect each other

$$OP = \frac{1}{3}\times 9 = 3\text{ cm}$$

$$OQ = \frac{1}{3}\times 9 = 3\text{ cm}$$

$$\therefore \quad PQ = 6\text{ cm}$$

14.



$$OB^2 + OC^2 = BC^2$$

$$OC^2 + OD^2 = CD^2$$

$$OD^2 + OD^2 = CD^2$$

$$OD^2 + OA^2 = AD^2$$

$$OA^2 + OB^2 = AB^2$$

$$\therefore 2(OB^2 + OA^2 + OD^2 + OC^2)$$

$$= AB^2 + BC^2 + CD^2 + DA^2$$

$$\Rightarrow \quad 2(AB^2 + CD^2) = AB^2 + BC^2 + CD^2 + DA^2$$

$$\Rightarrow \quad AB^2 + CD^2 = BC^2 + DA^2$$

Mensuration Solutions

1. (A)

(1) Side of one square $= \frac{40}{4} = 10$ cm.

[$\because$ Perimeter = 4 × side]

Side of other square $= \frac{32}{4}$

= 8 cm.

According to the question,

Area of third square

$= (10)^2 - (8)^2 = 100 - 64$

= 36 sq.cm.

Side of third square $= \sqrt{36}$

= 6 cm.

Its perimeter = 4 × 6 = 24 cm.

2. (C) Sides of the squares are 6 cm. 8 cm, 10 cm, 19 cm and 20 cm respectively.

Sum of their areas $= (6^2 + 8^2 + 10^2 + 19^2 + 20^2)$ cm²

$=(36+64+100 + 361 + 400)$ cm²

= 961 cm²

$\therefore$ Area of largest other square

= 961 cm²

$\therefore$ Its side $= \sqrt{961} = 31$ cm

$\therefore$ Required perimeter

= 4 × 31 = 124 cm.

3. (b) Let the side of square be a units. Area of this square $=a^2$

The diagonal of square

$= \sqrt{2}\,a$

$\therefore$ Area of square $= 2a^2$

$\therefore$ Required ratio $= a^2 : 2a^2$

= 1 : 2

4. (b)

(2) Side of the first square

$= \frac{40}{4} = 10$ cm

Side of the second square

$= \frac{24}{4} = 6$ cm

Difference of the areas of these squares

= (10 × 10 – 6 × 6) cm²

= (100 – 36) cm²

= 64 cm²

$\therefore$ Area of the third square

= 64 cm²

$\therefore$ Side of third square

$= \sqrt{64} = 8$ cm

$\therefore$ Perimeter of this square

= (4 × 8) cm

= 32 cm

5. (a)

Area of the rectangular garden

$=12\times5=60$ m²

=Area of the square garden

So that side of the square garden

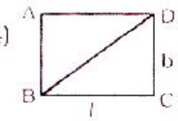
$= = \sqrt{60} m^2$

$= \sqrt{2} \times side$

$= \sqrt{2} \times \sqrt{60} = \sqrt{120} = \sqrt{14\times30} = 2\sqrt{30} cm$

6. (d)

(4)



BD = length of diagonal

= speed × time

$= \frac{52}{60} \times 15 = 13$ metre

$= \sqrt{l^2 + b^2}$

$\Rightarrow l^2 + b^2 = 169$

Again,

$(l + b) = \frac{68}{60} \times 15 = 17$ ...(ii)

$\therefore (l + b)^2 = l^2 + b^2 + 2lb$

$\Rightarrow 17^2 = 169 + 2\,l\,b$

$\Rightarrow 2\,l\,b = 289 - 169 = 120$

$\Rightarrow l\,b = \frac{120}{2} = 60$ m²

7. (a)

Area of garden without street

= 200 × 180 = 36000 sq.metre

Area of garden with street

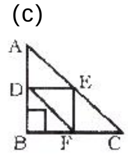
= 220 × 200 = 44000 sq.metre

$\therefore$ Area of the path

= 44000 – 36000

= 8000 sq.metre

8. (c)



$3^2 + 4^2 = 5^2$

Δ ABC is a right angled tria

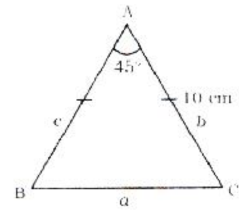
$\therefore$ ABC $= \frac{1}{2}$ × AB × BC

$= \frac{1}{2} \times 3 \times 4 = 6$cm²

$\therefore$ Required Area of Δ DEF

$= \frac{1}{4} \times 6 = \frac{3}{2}$ sq.cm.

9. (c)



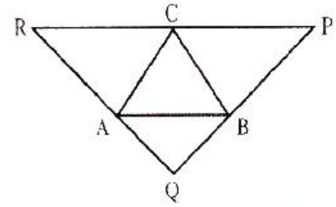
AB = AC = 10 cm

$\therefore$ Area $= \frac{1}{2}$ bc sin A

$= \frac{1}{2} \times 10\times10 \sin 45°$

$= \frac{50}{\sqrt{2}} = \frac{50\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = 25\sqrt{2}$ cm²

10. (c)



AQ || CB ,and AC || QB

$\therefore$ AQBC, is a parallelogram

$\Rightarrow$ BC = AQ

Again, AR || BC and AB || RC

$\therefore$ ARCB ,is a parallelogram.

$\Rightarrow$ BC = AR

$\Rightarrow$ AQ = AR

$\Rightarrow$ AQ = AR $= \frac{1}{2}$QR

$\Rightarrow$ BC $= \frac{1}{2}$QR

Similarly, AB $= \frac{1}{2}$PR and

AC $= \frac{1}{2}$PQ

$\therefore$ Required ratio

11. (c) Semi perimeter(s) =9+10+11/2=15 cm.

Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c}$

$= \sqrt{15(15-9)(15-10)(15}$

$= \sqrt{15\times6\times5\times4}$

$= 30\sqrt{2}$ square cm.

12. (a)

(1) Let the sides of triangle be a, b and c respectively.

$\therefore 2s = a + b + c = 32$

$\Rightarrow 11 + b + c = 32$

$\Rightarrow b + c = 32 - 11 = 21$ .....(i)

and $b - c = 5$ ......(ii)

By adding equations (i) and (ii)

$2b = 26 \Rightarrow b = 13$

$\therefore c = 13 - 5 = 8$

$\therefore 2s = 32 \Rightarrow s = 16$

$a = 11,\ b = 13,\ c = 8$

$\therefore$ Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

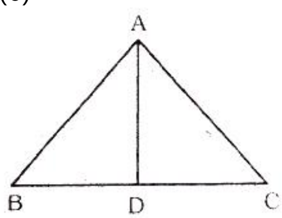
$= \sqrt{16(16-11)(16-13)(16-8)}$

$= \sqrt{16 \times 5 \times 3 \times 8}$

$= 8\sqrt{30}$ sq. cm.

v

13. (c)



AB = BC = CA = $2a$ cm,
AD $\perp$ BC

AD = $\sqrt{AB^2 - BD^2}$

$= \sqrt{4a^2 - a^2} = \sqrt{3}\,a$

$\therefore \sqrt{3}\,a = 15$

$\Rightarrow a = 5\sqrt{3}$

$\therefore 2a =$ Side $= 10\sqrt{3}$ cm

$\therefore$ Area of triangle

$= \frac{\sqrt{3}}{4} \times \left(10\sqrt{3}\right)^2$

$= 75\sqrt{3}$ sq. cm.

14. (c)

$15^2 + 20^2 = 25^2$

$\therefore$ The triangular field is right angled.

$\therefore$ Area of the field

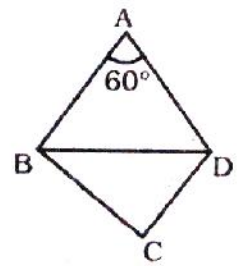
$= \frac{1}{2} \times 15 \times 20$

= 150 sq. metre

$\therefore$ Cost of sowing seeds

= 150 × 5 = Rs. 750

15. (a)



Side = $\frac{40}{4}$ = 10 cm

AB = AD = 10 cm

$\angle ABD = \angle ADB = 60°$

$\therefore$ Area of the rhombus

$= 2 \times \frac{\sqrt{3}}{4} \times (AB)^2$

$= 2 \times \frac{\sqrt{3}}{4} \times 10 \times 10$

$= 50\sqrt{3}$ cm²

16. (a)

17. (b) Let the sides of parallelogram be 5x and 4x

Base x Height = Area of parallelogram

= Area of parallelogram

$\therefore 5x \times 20 = 1000$

$\Rightarrow x = \frac{1000}{5 \times 20} = 10$

$\therefore$ Sides = 50 and 40 units

$\therefore 40x\ h = 1000$

$\Rightarrow h = \frac{1000}{40} = 25$ units

18. (b)

Area of the parallelogram

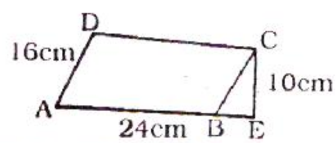
= Base × Height

= 15 × 12 = 180 sq.cm.

$\therefore$ 180 = 18 × height

$\Rightarrow$ Height = 10 cm

19. (d)



Area of the parallelogram

= Base × Height

= 24 × 10 = 240 sq.cm.

If the required distance be x cm, then

240 = 16 × x

$\Rightarrow x = \frac{240}{16} = 15$ cm

20. (a)

Diagonal of cube $= \sqrt{3a^2}$

$\therefore$ According to question

$\sqrt{3}\,a = 2\sqrt{3}$

$\Rightarrow a = 2$

$\therefore$ Its volume $= a^3 = 2^3$

= 8 cu cm

21. (c) Required answer

=volume of larger cube/volume of smaller

$= \frac{(15)^3}{(3)^3} = \frac{15 \times 15 \times 15}{3 \times 3 \times 3}$

$= 5 \times 5 \times 5 = 125$

22. (c) Diagonal of a cube

$= \sqrt{3} \times$ side

$4\sqrt{3} = \sqrt{3} \times$ side

$\therefore$ Side = 4 cm

$\therefore$ Volume of the cube

= (side)³ = (4)³ = 64 cm³

23. (c) Let the side of the two cubes are x and y.

According to the question.

$\frac{x^3}{y^3} = \frac{27}{64} = \frac{(3)^3}{(4)^3} \therefore \frac{x}{y} = \frac{3}{4}$

We know that surface area of the cube

= 6 × (side)²

$\therefore$ Ratio of their surface areas

$= \frac{6x^2}{6y^2} = \frac{6 \times 3^2}{6 \times 4^2} = \frac{9}{16} = 9 : 16$

24. (a) The length of the longest rod = The diagonal of the hall.

$= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{10^2 + 6^2 + 4^2}$

$= \sqrt{100 + 36 + 16} = \sqrt{152}$

$= \sqrt{2 \times 2 \times 38} = 2\sqrt{38}$ m

25. (b) We have

2×volume of cube = volume of cuboid

$\Rightarrow 2 \times (\text{edge})^3 = 9 \times 8 \times 6$ cu.cm.

$\Rightarrow (\text{edge})^3 = 9 \times 8 \times 3$

$\Rightarrow$ Edge $= \sqrt[3]{3 \times 3 \times 3 \times 2 \times 2 \times 2}$

= 3 × 2 = 6 cm.

$\Rightarrow$ Total surface area of the cube

= 6 × (edge)²

= 6 × 6 × 6 216 cm².

26. (d) Length of largest bamboo

$= \sqrt{(5)^2 + (4)^2 + (3)^2}$

$= \sqrt{25 + 16 + 9} = \sqrt{50}$

$= \sqrt{25 \times 2} = 5\sqrt{2}$ m

27. (b) Let the length of tank = x dm

Depth =x/3 dm

$$\text{Breadth} = \left(x - \frac{x}{3}\right) \times \frac{1}{3} \times \frac{1}{2}$$

$$= \frac{2x}{3} \times \frac{1}{3} \times \frac{1}{2} = \frac{x}{9}\text{ dm}$$

Volume of tank

$$= x \times \frac{x}{9} \times \frac{x}{3} = \frac{x^3}{27}$$

According to the ques-tion,

$$\frac{x^3}{27} = 216$$

$\Rightarrow x^3 = 27 \times 216$

$\Rightarrow x = (27 \times 216)^{1/3}$

$= 3 \times 6 = 18$ dm

28. (a) The external dimensions of the box are :

Length =20 cm, Breadth =12 cm. Height=10 cm

External volume of the box $20 \times 12 \times 10 = 2400\text{ cm}^3$

Thickness of the wood =1 cm

Internal length =20-2=18 cm

Internal breadth =12-2=10 cm

Internal height =10-2=8 cm

Internal volume $=18 \times 10 \times 8 = 1440\text{ cm}^3$

Volume of the wood $=(2400-1440)\text{ cm}^3 = 960\text{ cm}^3$

29. (d) surface area of a small cube

$= 6 \times (\text{edge})^2 = 6 \times 1 = 6\text{ cm}^2$

Surface area of the large cube $= 6(5)^2 = 6 \times 25\text{ cm}^2$

So that required ratio = $6/6 \times 25 = 1/25$, i.e., 1:25.

30. (b)

2) Let the length of the tank be x cm.

$$\therefore \text{Depth} = \frac{x}{3}$$

$$\text{Breadth} = \frac{1}{2} \times \frac{1}{3} \times \left(x - \frac{x}{3}\right)$$

$$= \frac{x}{9}$$

Now,

$$x \times \frac{x}{3} \times \frac{x}{9} = 216 \times 1000$$

$\Rightarrow x^3 = 27 \times 216 \times 1000$

$\Rightarrow x = (27 \times 216 \times 1000)^{1/3}$

$\Rightarrow x = 3 \times 6 \times 10$

$= 180$ cm $= 18$ dm

31. (b)

(2) Volume of cuboid

$= 9 \times 8 \times 6 = 432\text{ cm}^2$

According to the question,

Volume of cube

$$= \frac{432}{2} = 216\text{ cm}^3$$

$\therefore$ Edge of cube $= \sqrt[3]{216} = 6$ cm.

$\therefore$ Total surface area of cube

$= 6 \times (6)^2 = 216\text{ cm}^2$

32. (d) If the length of the edge of cube be x cm then

$$\text{diagonal} = \sqrt{3}x\text{ cm}$$

$\therefore \sqrt{3}x = 8\sqrt{3} \Rightarrow x = 8$ cm

$\therefore$ Surface area of the cube

$= 6x^2$

$= 6 \times 8 \times 8$

$= 384$ sq. cm

33. (b) Length of the longest pole

$= \sqrt{12^2 + 8^2 + 9^2}$

$= \sqrt{144 + 64 + 81} = \sqrt{289} = 17$

34. (C) Diagonal of the cube

$= 6\sqrt{3}$ cm

$\therefore \sqrt{3} \times \text{edge} = 6\sqrt{3}$ cm

$\Rightarrow$ Edge $= 6$ cm

$\therefore$ Total surface area : Volume

$= 6 \times 6^2 : 6^3 = 1 : 1$

35. (b)

Length of the edge of the box

$= \sqrt[3]{3.375}$

$= \sqrt[3]{1.5 \times 1.5 \times 1.5} = 1.5$ Meter

36. (d) Area of the floor =volume of room/Height of room

=204/6=34 sq. m.

37. (c)

(3) Volume of cylindrical vessel = $\pi r^2 h$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\therefore \text{Number of cones} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = 3$$

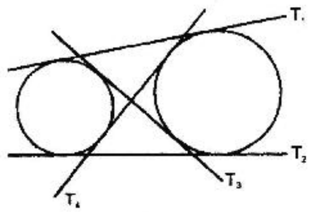
Practice Set-2 Solution

1. Circumcentre of a triangle is the point of intersection of perpendicular bisectors of its sides.
2. Let height of Qutab Minar be x metres.

$$\therefore \quad \frac{150}{120} = \frac{x}{80}$$

$$\Rightarrow \quad x = \frac{150 \times 80}{120} = 100$$

3. By following figur, Hence 4 tangents can be drawn.



4.

Let the angle be $x°$.

$$\therefore \quad x = \frac{1}{3} \times (180 - x);$$

Hence $\quad x = 45°$

5.

6. Sum of the interior angles of a polygon of n sides

$$= (2n - 4) \times \frac{\pi}{2}$$

$$\therefore \quad (2n - 4) \times \frac{\pi}{2} = 1620 \times \frac{\pi}{180}$$

$$\Rightarrow \quad n - 2 = 9$$

$$\Rightarrow \quad n = 11$$

7.

Let n be the number of sides of the polygon

$\therefore$ Interior angle = 8 × Exterior angle

$$\Rightarrow \quad \frac{(2n-4) \times \frac{\pi}{2}}{n} = 8 \times \frac{2\pi}{n}$$

$$\Rightarrow \quad n - 2 = 16$$

$$\Rightarrow \quad n = 18$$

8.

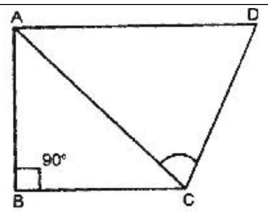
$\frac{5\pi}{6} = 150°.$

Exterior angle = 30°

$$\therefore \quad \text{Number of sides} = \frac{360°}{\text{Exterior angle}}$$

$$= \frac{360°}{30°} = 12.$$

9.



$$AD^2 = AB^2 + BC^2 + CD^2$$

$$= AC^2 + CD^2$$

$$\Rightarrow \quad \angle ACD = 90°$$

10.

$x + y + (y + 20) = 180$

$\Rightarrow \quad x + 2y = 160 \quad (i)$

and $\quad 4x - y = 10 \quad (ii)$

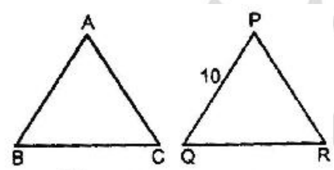
Solving (i) and (ii) we get

$\Rightarrow \quad y = 70, x = 20$

Hence the angles of the triangle are 20°, 70°, 90°.

Thus, the triangle is right angled.

11.



$$\therefore \quad \frac{AB}{PQ} = \frac{36}{24}$$

$$\Rightarrow \quad AB = \frac{3}{2}$$

$$PQ = \frac{3}{2} \times 10 = 15$$

12.

$AB = BC$

$$\Rightarrow \quad \angle BAC = \angle BCA = \frac{110°}{2} = 55°$$

$\therefore \quad \angle ACD = 125°$

Also, $\quad AC = CD$

$$\Rightarrow \quad \angle CAD = \angle ADC = \frac{55°}{2} = 27.5°$$

13.

Given triangles are congruent.

$$\therefore \quad \frac{\text{Area of Ist triangle}}{\text{Area of IInd triangle}} = \frac{h_1^2}{h_2^2},$$

$$\Rightarrow \quad \frac{h_1^2}{h_2^2} = \frac{9}{16}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{3}{4}$$

14.

$AB = 25$

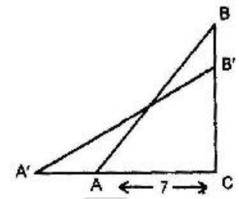
$\Rightarrow \quad BC = 24$

$\Rightarrow \quad B'C = 20$

$A'B' = 25$

$\Rightarrow \quad A'C = 15$

$\Rightarrow \quad A'A = 8$ m



15.

Area of trapezium

$$= \frac{1}{2} \text{(sum of parallel sides)} \times \text{height}$$

$$= \frac{1}{2}(5x + 3x) \times 24 = 1440.$$

Solving, we get $\quad x = 15$

and $\quad$ length of longer side $= 5 \times 15 = 75.$

16.

Degrees at 3 : 20

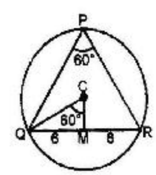
$$= 5 \text{ min distance} - \frac{1}{3} \text{(5 min) distance}$$

$$= 30 - 10 = 20 \text{ degrees.}$$

17.

Circumcentre of a triangle is the point of intersection of the right bisectors of its sides.

$$\therefore \quad \sin 60° = \frac{QM}{QC}$$



$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{6}{QC}$$

$$\Rightarrow \quad QC = \frac{6 \times 2}{\sqrt{3}} = 4\sqrt{3} \text{ cm}$$

18.

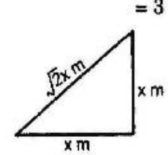
Perimeter $= (6 + 3\sqrt{2})$ m

i.e. $\quad x + x + \sqrt{2}x = 6 + 3\sqrt{2}$

$$\Rightarrow \quad 2x + \sqrt{2}x = 6 + 3\sqrt{2}$$

$$\Rightarrow \quad x = \frac{6 + 3\sqrt{2}}{2 + \sqrt{2}} = \frac{3(2 + \sqrt{2})}{2 + \sqrt{2}}$$

$$= 3$$



$\therefore$ Required area of triangle

$$= \frac{1}{2} \times x \times x = \frac{1}{2}x^2$$

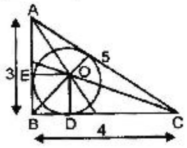
$$= \frac{1}{2} \times (3)^2 = 4.5 \text{ m}^2$$

19.

If incircle of a triangle ABC touches BC at D, then

$|BD - CD| = |AB - AC|$

In our case, AC = 5, AB = 3



$\Rightarrow \quad AC - AB = 2$

$\therefore \quad CD - BD = 2$

In our case, $BC = 4$

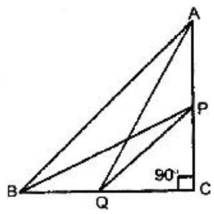
$\Rightarrow \quad BD + DC = 4$ and $-BD + DC = 2$

$\Rightarrow \quad CD = 3$

$\Rightarrow \quad BD = 1 = OE$

= Radius of the incircle

20.



$AQ^2 = AC^2 + QC^2$

$BP^2 = BC^2 + CP^2$

$AQ^2 + BP^2 = (AC^2 + BC^2) + (QC^2 + CP^2)$

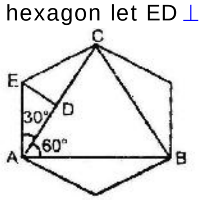
$= AB^2 + PQ^2$

$= AB^2\left(\frac{1}{2}AB\right)^2, \left[\because PQ = \frac{1}{2}AB\right]$

$= \frac{5}{4}AB^2$

$4(AQ^2 + BP^2) = 5AB^2$

21. Let a be the side of th regular hexagon let ED$\perp$ AC



$\therefore \quad AD = \frac{p}{6} = CD$

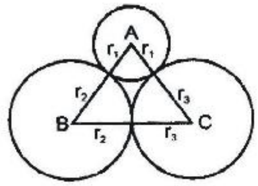
From ΔADE, $\frac{AD}{AE} = \cos 30°$

$\Rightarrow \quad \frac{\frac{p}{6}}{a} = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad 6a = \frac{2p}{\sqrt{3}}$

$\therefore$ Perimeter of regular hexagon $= \frac{2p}{\sqrt{3}}$

22.



$AB = r_1 + r_2 = 4$

$BC = r_2 + r_3 = 6$

$CA = r_3 + r_1 = 8$

$\therefore \quad 2(r_1 + r_2 + r_3) = 4 + 6 + 8$

$\Rightarrow \quad r_1 + r_2 + r_3 = 9$

23.

$\angle OBC = 37°$

$OB = OC =$ radius

$\therefore \quad \angle OCB = \angle OBC = 37°$

$\therefore \quad \angle BOC = 180° - (37° + 37°) = 106°$

Now, $\angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 106 = 53°$

24.

From the given figure,

$PC \times PD = PA \times PB$

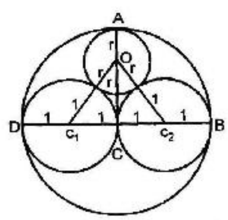
$\therefore \quad (4 + 3) \times 4 = 8 \times PB$

$\Rightarrow \quad PB = \frac{28}{8} = 3.5$ cm

$\therefore \quad AB = AP - BP = 8 - 3.5$

$= 4.5$ cm.

25.



$CC_1 = 1, OC_1 = r + 1$

$OC = AC - AO = CD - AO,$

[AC and CD are radius of the bigger circle]

$\Rightarrow \quad OC = 2 - r$

$OC^2 = CC^2 + OC^2$

$(r+1)^2 = 12 + (2-r)^2$

$r^2 + 2r + 1 = 1 + 4 - 4r + r^2$

$6r = 4 \Rightarrow r = 4/6 = 2/3$

Explanation Advance Mensuration

1. Side of regular hexagon

$= 2a$ cm

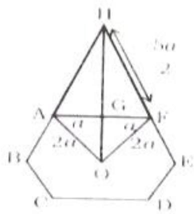
area of hexagon $= 6 \times \frac{\sqrt{3}}{4} \times (2a)^2$

$\Rightarrow \quad = 6\sqrt{3}a^2 \text{ cm}^2$

Slant edge of pyramid

$\Rightarrow \quad = \frac{5a}{2}$ cm

$\therefore$



slant edge $\Rightarrow \frac{5a}{2}$

(Given)

$\Rightarrow \quad HF = \frac{5a}{2}$ (slant height)

$\Rightarrow \quad HG =$ slant height (1)

$\Rightarrow \quad GF =$ base

$\Rightarrow \quad$ (a) given

slant height $\Rightarrow \sqrt{\left(\frac{5a^2}{2}\right) - (a)^2}$

$= \sqrt{\frac{25a^2}{4} - a^2} = \frac{\sqrt{21}a}{2}$

AOF is equilateral triangle of side $2a$

$\therefore$ Altitude $\quad GO = \frac{\sqrt{3}}{2} \times 2a$

$= \sqrt{3}a$

$\therefore$ Slant height $= \frac{\sqrt{21}}{2}a$

altitude $= \sqrt{3}a$

$\therefore$ Height of the pyramid $= h$

$\Rightarrow \quad \sqrt{\left(\frac{21a}{2}\right)^2 - (\sqrt{3}a)}$

$= \sqrt{\frac{21}{4}a^2 - 3a^2}$

$= \sqrt{\frac{92^2}{4}}$

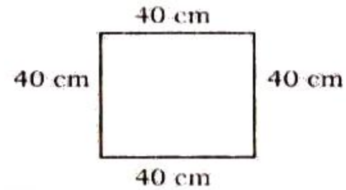
$\therefore$ Volume of pyramid

$= \frac{1}{3}$ area of base $\times$ height

$= \frac{1}{3} \times 6\sqrt{3}a^2 \times \frac{3}{2}a$

$= 3\sqrt{3}a^3 \text{ cm}^3$

2.



$\Rightarrow$ area of base

$= 40 \times 40$

$= 1600 \text{ cm}^2$

Let height of pyramid $= h$

$\therefore$ volume $= \frac{1}{3} \times h \times$ area of base

$= \frac{1}{3} \times h \times 1600$

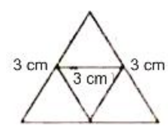
$= 8000$ (given)

$= h = 15$ cm

3. Edge of regular tetra hadron

$= 3$ cm

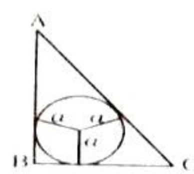
$\therefore \quad a = 3$ cm



$\therefore$ volume $\Rightarrow \frac{\sqrt{2}}{12}a^3 \text{cm}^3$

$\Rightarrow \frac{\sqrt{2}}{12} \times (3)^3 \qquad = \frac{9}{4}\sqrt{2} \text{ cm}^3$

4.



r- inradius of incircle of triangle

Perimeter $= 15$ cm (given)

$\therefore$ Semiperimeter(S) $= \frac{15}{2}$ cm

Indradius of any triangle

$r \Rightarrow \frac{\Delta}{s}$

Where Δ is the area of triangle

$\therefore \quad r = 3$ cm given

$3 \Rightarrow \frac{\text{area of triangle}}{\frac{15}{2}}$

$3 \times \frac{15}{2} =$ area of triangle

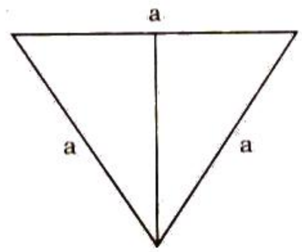
$\Rightarrow \quad \frac{45}{2}\text{cm} = \text{are of triangle}$

$\therefore$ volume of prism $\Rightarrow 270\ \text{cm}^3$ (given)

$\therefore \quad 270 = h \times \frac{45}{2}$

$\Rightarrow \quad h = 12\ \text{cm}$

5.



Let side equilateral triangle be

$= a$

$\therefore$ area $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}a^2 = 173\ \text{cm}^2$

$\Rightarrow \quad a^2 = \frac{173}{\sqrt{3}} \times 4$

$(\sqrt{3} = 1.73)$

$\therefore \quad a^2 = \frac{173}{1.73} \times 4$

$= \frac{173}{173} \times 4 \times 100$

$a^2 = 400$

$a = 20\ \text{cm}$

Perimeter of base $= 20 \times 3$

$= 60\ \text{cm}$

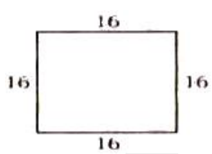
$\therefore$ Volume of prism $\times 10380\ \text{cm}^3$ (given)

area of base$\times$ height

$\text{height} = \frac{10380}{173} = 60$

LSA $= 60 \times 60 = 3600\ \text{cm}^2$

6.



#Perimeter of the base $= 4 \times 6$

$= 64$

Curved or lateral surface are of pyramid

$= \frac{1}{2} \times (\text{perimeter of base}) \times$ height

figr

$\Rightarrow$ height of pyramid $\Rightarrow 15$ cm

$\Rightarrow$ base $= 8$ cm

$\Rightarrow$Slant height of pyramid

$l = \sqrt{(15)^2 + (8)^2}$

$\Rightarrow \quad = 17$ cm

$\Rightarrow$ curved surface are of pyramid

$\Rightarrow \frac{1}{2} \times 64 \times 17 \Rightarrow 544\ \text{cm}^3$

7. Volmue of pyramid

$= \frac{1}{3} \times \text{Area of base} \times \text{height}$

$= \frac{1}{3} \times 57 \times 10 = 190\ \text{cm}^3$

8. Volume of prism $= \frac{\sqrt{3}}{4}\boldsymbol{a^2 \times h}$

$= \frac{\sqrt{3}}{4} \times (8)^2 \times 10$

$= 160\sqrt{3}\ \text{cm}^3$

9. Total slant surface area

$= 4 \times \frac{1}{4} \times a = 12$

(where a is the side of the square base)

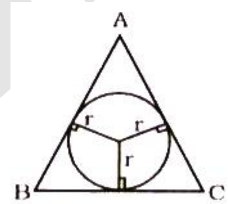
$\Rightarrow \quad a = \frac{12}{8} = \frac{3}{2}$ cm

$\Rightarrow$ area of base

$= \frac{9}{4}\ \text{cm}^2$

$\therefore$ Required ratio $= \frac{12}{\frac{9}{4}} = 16:3$

10.



In radius of triangle $= \frac{\text{area of triangle}}{\text{semiperimeter}}$

$\therefore ar(\Delta ABC)$ =Inradius $\times$ semiperimeter

$= 4 \times \frac{28}{2} = 4 \times 14 = 56$ cm

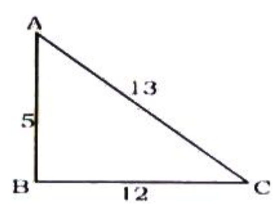
Volume of the prism $= 366\ \text{cm}^3$

(area of base)$\times$ height $= 366\ \text{cm}^3$

$56 \times \text{height} = 366$ cm

$\text{height} = \frac{366}{56} = 6.535$ cm

11.



Clearly the base triangle is the right triangle)

$\therefore$area of triangle ABC $= \frac{1}{2} \times 5 \times 12$

$= 30\ \text{cm}^2$

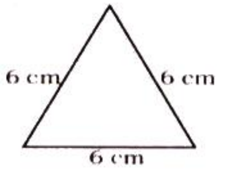
Volume of the pyramid $= \frac{1}{3} \times (\text{base area}) \times \text{height}$

$\frac{1}{3}\times$Base area$\times$ height $= 330$

$\frac{1}{3}\times 30\times$height $= 330$

height $= \frac{330\times 3}{30} = 33$ cm

12.



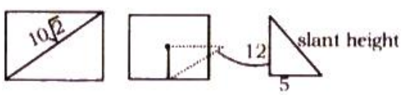
Volume of prims $=$ area $\times$ height

$= \frac{\sqrt{3}}{2}(6)^2 \times$ height

$\frac{\sqrt{3}}{4}\times 6\times 6\times$height $= 81\sqrt{3}$

height $= \frac{81\sqrt{3}\times 4}{\sqrt{3}\times 6\times 6} = 9$ cm

13.



Side of square $= \frac{1}{\sqrt{2}}\times 10\sqrt{2} = 10$ cm

Slant height $= \sqrt{5^2+12^2} = 13$ cm

lateral surface area $= \frac{1}{2}\times$perimeter of base

slant height $= \frac{1}{2}\times 40\times 13$

$= 260\ \text{cm}^2$

14. Total surface are of prism

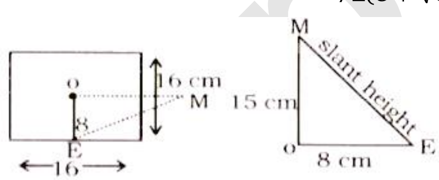
$=$(perimeter of base$\times$height+2$\times$base area)

$= (8\times 12\times 10) + 2\times\frac{\sqrt{3}}{4}\times 12^2$

$= 360 + 72\sqrt{3}$

$= 72(5+\sqrt{3})\ \text{cm}^2$

15.



Slant height of pyramid $= \sqrt{8^2+15^2} = 17$

(8,15,17) is triplet)

lateral surface are $= \frac{1}{2}\times$perimeter of base$\times$

slant height

$= \frac{1}{2}\times 64\times 17$

$= 32\times 17 = 544\ \text{cm}^2$

16. Height of pyramid = 6 m

Diagonal of square base $= 24\sqrt{2}$ m

Side of square $= 24$ m

Area of square $= (24)^2$

$= 576\ \text{m}^2$

Volume of the pyramid $= \frac{1}{3}\times$area of base$\times$height

$= \frac{1}{3}\times 576\times 6$

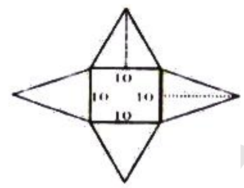
$= 576\times 2 = 1152\ \text{m}^3$

17. Total surface area

$= 4\times\left[\frac{\sqrt{3}}{4}\times 1^2\right]$

$= \sqrt{3}\ \text{cm}^2$

18.



Area of base $= 10\times 10 = 100\ \text{cm}^2$

Area of 4 base $= (\frac{1}{2}\times$base$\times$slant height$)\times 4$

$\Rightarrow (\frac{1}{2}\times 10\times 13)\times 4$

$= 65\times 4 = 260$

(Slant height $= \sqrt{12^2+5^2} = \sqrt{169} = 13$)

Total Surface area

$\Rightarrow 260 + 100$

$\Rightarrow 360\text{m}^2$

19. Let the side of the square = *a* cm

ATQ

T.S.A.=C.S.A+2 base area

C.S.A.= base perimeter$\times h$

Volume =base area$\times h$

$\therefore$ T.S.A. =base perimeter$\times h$ + 2 base area

$192 = 4a\times 10 + 2a^2$

$2a^2 + 40a - 192 = 0$

$a^2 + 20a - 96 = 0$

$a(a+24) - 4(a+24) = 0$

$(a+24)(a-4) = 0$

$\therefore\quad a = 4, (-24)$

$\therefore\quad a = 4$ (side can never be in $-$ve)

Volume $=$ base area $\times\ h$

Volume $= 16\times 10$

Volume $= 160\ \text{cm}^3$

20. Seme-perimeter of

$\Delta = \frac{7+8+9}{2} = 12$ cm

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(17-7)(12-8)(12-9)}$

$= \sqrt{12\times 5\times 4\times 3}$

$= 12\sqrt{5}$

21. According to the question

$$\text{Volume of cylinder} = \Pi r^2 h$$
$$\text{Volume of Sphere} = \frac{4}{3}\Pi r^3$$

The number of spherical balls

$$= \frac{\text{P}r^2 \text{ h}}{\frac{4}{3}\text{P}h^3}$$
$$= \frac{30 \times 30 \times 40 \times 3}{4 \times 1 \times 1 \times 1} = 27000$$

22. According to the question

Volume of cylinder = Volume of cone

$$\pi r^2 h_1 = \frac{1}{3}\pi r^2 h_2$$
$$\frac{h_1}{h_2} = \frac{1}{3}$$

23. Total surface area of prism = perimeter of Base × Height + 2 × Base Area

$$10 = 4a \times 2 + 2 \times a^2$$
$$10 = 8a + 2a^2$$
$$a^2 + 4a - 5 = 0$$
$$(a+5)(a-1) = 0$$
$$a = 1, a = -5$$

Volume of Prism = Area of base × height

$$= 1 \times 1 \times 2 = 2\,\text{cm}^3$$

24. Let the radius of wire = 1 cm

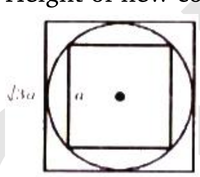
$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(1)^2 \text{h} = \frac{1}{3}\pi h$$
$$\text{New radius of wire} = \frac{1}{3}\text{cm}$$
$$\text{Volume of new cone} = \frac{1}{3}\pi\left(\frac{1}{3}\right)^2 \text{ H}$$

Volume of old cone = Volume of new cone

$$\frac{1}{3}\pi \text{ h} = \frac{1}{27}\pi \text{ H}$$

Height of new cone is increased by 9 times.

25.



$$\text{Volume of small cube} = a^3$$
$$\text{Length of big cube} = \sqrt{3}a$$
$$\text{Volume of big cube} = (\sqrt{3}a)^3$$
$$= 3\sqrt{3}a^3$$
$$V_1 : V_2 = a^3 : 3\sqrt{3}a^3 = 1 : 3\sqrt{3}$$

26. $\mathbf{\frac{4}{3}\pi r^3 = \pi r^2 h}$

$$\frac{4}{3} \times 6 = h$$
$$h = 8\,\text{cm}$$

Curved surface area of cylinder

$$= 2\pi rh = 2 \times \pi \times 6 \times 8$$
$$= 96\pi\,\text{cm}^2$$

27. $\boldsymbol{y + b + h = 19}$

$$d = \sqrt{l^2 + b^2 + h^2} = 5\sqrt{5}$$
$$l^2 + b^2 + h^2 = (5\sqrt{5})^2 = 125$$

Now, $(l+b+h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)$

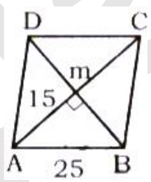
$$(19)^2 = 125 + 2(lb + bh + hl)$$
$$361 - 125 = 2(lb + bh + hl)$$
$$2(lb + bh + hl) = 236$$

So, Total surface area = $236\,\text{cm}^2$

Alternate :

$$(19)^2 - (5\sqrt{5})^2 = 361 - 125$$
$$= 236\,\text{cm}^2$$

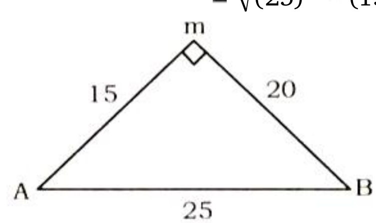
28.



ABCD is a Rhombus

(Diagonal of Rhombis bisects eachother at Right angle)

$$mB = \sqrt{(AB)^2 - (mA)^2}$$
$$= \sqrt{(25)^2 - (15)^2} = 20\,\text{cm}$$

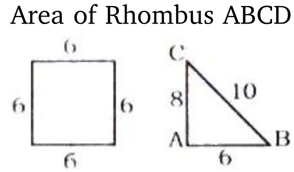


Area of Δ MAB = $\frac{1}{2} \times 15 \times 20 = 150$

∵ Rhombus has 4 equal Triangles

Area of Rhombus ABCD = $4 \times 150 = 600\ \text{cm}^2$

29.



Side of square = $\frac{24}{4} = 6$

then area of $\Delta ABC = \frac{1}{2} \times 8 \times 6 = 24$

DATA INTERPRETATION

1. TABLE

Directions (1-4):

1. (C) Average = $\frac{476}{6} = 79.3 \sim \boxed{80}$

2. (d) (P+Q) (R+S)

1980 – 24 24
1990- 30 23
1991 – 30 26
1994 – 20 39
1993- $\boxed{33 \quad 33}$

So, None of above (i.e. 1993 has equal production)

3. (d) Clearly, it is visible , S type was in continous increase

4. (a) 25 % of 80 = 20

Direction (5-9) :

5. (b) Average loan = $\frac{87+104+113+120}{4} = \frac{424}{4}$

$\Rightarrow$ Rs 106 crore

Required year is 1996 where loan disbursed is Rs. 104 crore

6. (b) Percentage increase = $\frac{120-113}{113} \times 100$

$= \frac{700}{113} = \boxed{6\frac{22}{113}\%}$

7. (d) (A+B) (C+D)

1995 – 45 42
1996 – 56 48
1997 – 63 50
1998 – 71 49

None of above

8. (d) Clearly, bank D has continued increase in loan disbursement

9. (b) Year 1998 $\rightarrow$ 30% of 120

$\Rightarrow$ Rs. 36 crore

B has Rs. 41 crore disbursement of loans.

Direction (10-14):

10. (a) out of given option, age group (16-25) has maximum population.

11. (a) below 26 years =(30+17.765)% =47.75%

$\frac{47.75 \times 4200}{100} = 2005.5 = 2006$

12. (b) Percent of people(Below 36 years) =65%

$\frac{200 \times 512\%}{465\%}$

= 15.75 millions

13. (b) Percent of people (above 56 years) =6.25%

6.25% $\rightarrow$ 10 millions

Different between (16-25) of (46-55) =3.5%

6.25% $\rightarrow$ 10 millions

3.5% $\rightarrow$ $\frac{10}{6.25} \times 3.5 = 5.6$ millions.

14. (b) Percent difference (46-55) and (26-35) age group is =3%

Now 3% $\rightarrow$ 11.75 millions

Total population (100%) $\rightarrow$ $\frac{11.75}{3} \times 100$

=391.67 millions.

Question 15-17

15.

(2) Total marks of Charu = 72% of 100 + 60% of 100 + 68% of 150 + 74% of 60 + 68% of 150 + 75% of 40

i. = 72 + 60 + 102 + 44.4 + 102 + 30 = 410.4

percentage of marks = $\frac{410.4}{600} \times 600$

= 69 approx.

16.

(2) Required percentage = $\frac{55\% \text{ of } 40}{66\% \text{ of } 100} \times 100$

= 33.33%

17.

(2) Required percentage = $\frac{80\% \text{ of } 60 + 62\% \text{ of } 40}{60+40}$

$\times$ 100 = 72.8

$\times \frac{4}{15}$ = 30696, ; reqd. % = $\frac{4686}{30696} \times 100$

= 15% approx.

(18-21) :

18. 2; Average number of candidates appeared for

State B = $\frac{6400+7800+7000+8800+9500}{5}$

$= \frac{39500}{5} = 7900$

19. 1; Total number of candidates selected for all the states together in the year 1996

= 80 + 70 + 48 + 85 + 78 = 361

Total number of candidates qualified for all the states together in the year 1996

= 950 + 650 + 400 + 620 + 720 = 3340

$\therefore$ Required percentage

$= \frac{361}{3340} \times 100$

= 10.8% $\approx$ 11%

20. 4; Percentage of candidates selected for State C can be seen in the following table:

Percentage of candidates selected over the number of candidates qualified for different states in different years can be tabulated as shown below:

Years	Percentage of candidates selected to the qualified in State C.
1994	$\left(\frac{55}{350} \times 100\right) = 15.7$
1995	$\left(\frac{65}{525} \times 100\right) = 12.3$
1996	$\left(\frac{48}{400} \times 100\right) = 12$
1997	$\left(\frac{70}{560} \times 100\right) = 12.5$
1998	$\left(\frac{82}{640} \times 100\right) = 12.8$

Clearly, the required percentage is the highest for the year 1994.

Quicker Approach:

We have to find the year for which $\frac{\text{Selected}}{\text{Qualified}}$ is the highest;

i.e., $\frac{\text{Qualified}}{\text{Selected}}$ is the least.

Clearly, only for the year 1994 it is below 7. In others cases it is more than 7. Hence our answer is options (4).

21. 2; Average number of candidates selected over the years in different states can be tabulated as shown below:

States	**Average number of candidates selected over the years**
A	$\frac{75+60+80+75+70}{5}=72$
B	$\frac{60+84+70+86+90}{5}=78$
C	$\frac{55+65+48+70+82}{5}=64$
D	$\frac{75+70+85+65+48}{5}=68.6$
E	$\frac{75+85+78+82+94}{5}=82.8$

Clearly, the required state is E.

DATA INTERPRETATION

2. BAR GRAPH

Directions 1-5: The following bar chart shows the sales of a company XYZ (in Rs. Crore). Study the chart and answer the following questions.

1. (b) Total sales in 2nd and 3rd year
Rs. 1773+1115 = Rs. 2888 crore
2. (a) 10th, It is clear from the graph
3. (b) rd, it is clear from the graph
4. (b) Mean $= \frac{8730+924}{2} = \frac{9654}{2} = \text{Rs.}4827$
5. (b) Required difference =(5345-1841) =3504 crores
6. (d) Total accidents $= \frac{230}{1000} \times 100 = 23\%$

Percentage of accidents with two wheelers

And other subjects $= \frac{770 \times 100}{1000} = 77\%$

Required difference =7-23=54%

7. (c) Two wheelers + cars + Bases + stationary vehiclers

230+150+120+100=600

$\frac{600}{1000} \times 100 = 60\%$ Ans.

8. (d) 360º =1000

1º=1000/360º

36º $= \frac{1000}{360^\circ} \times 36^\circ$

9. (a) required percentage $= \frac{40+200}{1000} \times 100 = \frac{24000}{1000} = 24\%$
10. (b) required difference $= \frac{160-120}{1000} \times 100 = 4\%$

Directions 11-12 : The following bar diagram depicts figures for some agricultural imports from January May, 2008, Answer (as closely as possible) the questions using the date provided here.

11. (a) required average price

$= \frac{33 \times 120 + 33 \times 120}{2}$

$= \frac{120 \times 66}{2} = 3960$

12. (b) required csot of wheat $= 36 \times 156 = 5616$

Q. 13-16:

13. (d) 60+80/2=70
14. (c) 70+10/2=40
15. (c) 80+50+10+20/4=40.
16. (c) 60+50+70+30=210

(17-21) :

17. 4; Average value of imports in the years 1994, 1995 and 1997 $= \frac{250+220+280}{3} =$ Rs. 250 cr

$\therefore$ Required percentage $= \frac{450}{250} \times 100 = 180\%$

18. 4; Required percentage $= \frac{375}{250} \times 100 = 150\%$
19. 1; Average import

$= \frac{80+150+250+220+350+280}{6}$

$= \frac{1330}{6} \approx 222\,cr$

Average export

$= \frac{150+225+375+300+450+330}{6} = 350$ cr

$\therefore$ Required difference = 83 cr $\approx$ 85 cr

20. 2; It is obvious from the given graph.
21. 4; Required percentage increase

$= \frac{450-300}{300} \times 100 = \frac{150}{300} \times 100 = 50\%$

DATA INTERPRETATION

3. LINE GRAPH

Directions 1-5 :

1. (a) Both the lines in the graph intersect at 10:30 am

2. (b) average speed $=\frac{120}{5/2}=48$ km/h

3. (c) time =11:30-9:00 = $2\frac{1}{2}$ hours

4. (d) 80, it clear from the graph

5. (b) difference between temperature
Sunday =39-23=16°
Saturday =42.5-24=18.5° (maximum)
Wednesday =32.5-15=17.5°

6. (a) $\frac{\text{Exports}}{\text{imports}}=1.75=\frac{175}{100}=\frac{7}{4}$
After 40% increase imports
$\text{imports}=4\times\frac{140}{100}=\frac{560}{100}=\frac{56}{10}$

$\frac{\text{Exports}}{\text{Imports}}=\frac{7\times10}{56}=\frac{70}{56}=\frac{5}{4}=1.25$

7. (b) In the year 2005
Imports of company x=Rs. 180 crores
Exports =1.75×180=Rs. 315 crores
Exports of company y=Rs. 157.5 crores
Imports of company y =157.5/0.75=210 crores

8. 1; Number of students in 1994
= 1500 + (300 – 250) + (250 – 350)
= 1500 + 50 – 100 = 1450
Number of students in 1995
= 1450 + (500 – 400) = 1550
∴ Required increase = 1500 – 1400 = 100

9. 4; From the graph's inclination, it is clear that the percentage rise/fall is maximum in the year 1997 with respect to previous year.

10. 4; Number of students in 1996
= 1550 + (450 – 300) = 1700

11. 4; Strength of the school in different years

1993	1994	1995	1996	1997	1998
1550	1450	1550	1700	1600	1650

12. 2; Required % = $\frac{1700}{1450}\times100\approx117\%$

(13-17) :

13. 4; There is no relationship between the revenue expenditure in 1997-98 and 1996-97. So the total revenue expenditure in 1996-97 can't be determined.

14. 4; Without knowing the total expenditure for the two financial years, we can't find out the answer.

15. 1; Required revenue different between others and defence = (20 – 14)% of 302537 = 18152.22 crore

16. 3; Required percentage $=\frac{16}{36}\times100=44.45\%$

17. 2; Total revenue expenditure on grants to state and Uts
$=\frac{47781}{15}\times18.6\approx59250$ crore

(18-22) :

18. 2; ∴ Profit = Income – Expenditure
Profit = $\frac{\%\ \text{Profit}\times\text{Expenditure}}{100}$
Clearly, profit of the company will depend on the value of the (% Profit × Expenditure). Greater the value of this greater the amount of profit. By visual inspection of the graph we can see that the maximum amount of profit is in the year 2001.

19. 1; Income of the company in different years is as given below:
1996 = 80.50, 1997 = 108.90, 1998
= 175.50, 1999 = 150, 2000 = 210 and 2001
= 279
∴ Required average
$=\frac{80.50+108.90+175.50+150+210+279}{6}\approx$ Rs. 170 lakhs.

20. 2; The maximum difference in the % profit the company for any two consecutive years is 15 and the minimum base is 21. Hence, our answer is 1998.

21. 4; Income of company in 2000 = $=150\left(\frac{140}{100}\right)=210$

22. 1' Income in 1998 = 140% of 130 = Rs. 182 lakhs.

DATA INTERPRETATION

4. PIE CHART

Directions (1-5): Read the following pie- chart to answer the questions given below it.

1. (b) Amount spend on the food
=23% of 46,000
$46000\times\frac{23}{100}=10,580$

2. (a) clothing and housing =10+15=25%
$\frac{25}{100}\times 46000=11,500$

3. (d) Housing 15% and Education 12%
= 5:4

4. (a) Maximum amount is spent on food i.e., 23%

5. (a) saving =15% of 46,000
$\frac{15}{10}\times 46000=\text{Rs.}6900$

6. (c) 32% =800000
1% =800000/32
3% $=\frac{800000}{32}\times 3=\text{Rs.}75000$

7. (b) Required percentage = $\frac{3}{25}\times 100=12\%$

8. (a) required ratio =3:7 (from chart)

9 Income = Rs. 360000
Savings = $\frac{60}{360}\times 36000=\text{Rs.}60000$

10. Education – housing = 70° - 54° =16°=Rs. 1600
1°=Rs. 100
Expenditure on food =120° = 120×100=Rs. 12000

11. (a) Expenditure on food/savings =2/1

(12-16) :

12. 4; Aid received by Middle East & North Africa
$=\frac{6.4}{16}\times 21$ = Rs. 8.4 billion

Aid received by East Asia & Pacific $=\frac{6.4}{16}\times 15$
= Rs. 6 billion

∴ More aid = 8.4 – 6 = Rs. 2.4 billion

13. 2;

14. 1; Aid received by Sub-Saharan Africa
$=\frac{6}{10}\times 14$
= Rs. 14.43 billion

Aid received by East Asia & Pacific $=\frac{6}{10}\times 24$
= Rs. 8.4 billion

∴ Required % = $\frac{14.4-8.4}{8.4}\times 100=71.42\%$

15. 4; Aid received by South Asia in 2008
$=50\times\frac{9}{100}=$ Rs. 4.5 billion

Aid received by South Asia in 2013
$=45\times\frac{10}{100}$ = Rs. 4.5 billion

∴ Hence, the difference is zero.

16. 3; Aid received by all countries in 2008
$=\frac{10.5\times 100}{100}$ = Rs. 50 billion

Aid received by all the countries in 2013
$=\frac{2.4\times 100}{4}$ = Rs. 60 billion

∴ Required ratio = 5 : 6

(17-21) :

17. 3;

18. 1; Number of students appearing from Bihar in
2012 = 20% of 2.40 = 48000

Number of students appearing from WB in 2011
= 20% of 2.50 = 50000

Required % $=\frac{48000}{50000}\times 100=96\%$

19. 4;

20. 4; Required % $=\frac{30000}{240000}\times 100=12.50\%$

21. 2; Required percent $=\frac{18\text{ \% of }2.50}{23\text{\% of }2.40}\times 100\approx 80\%$